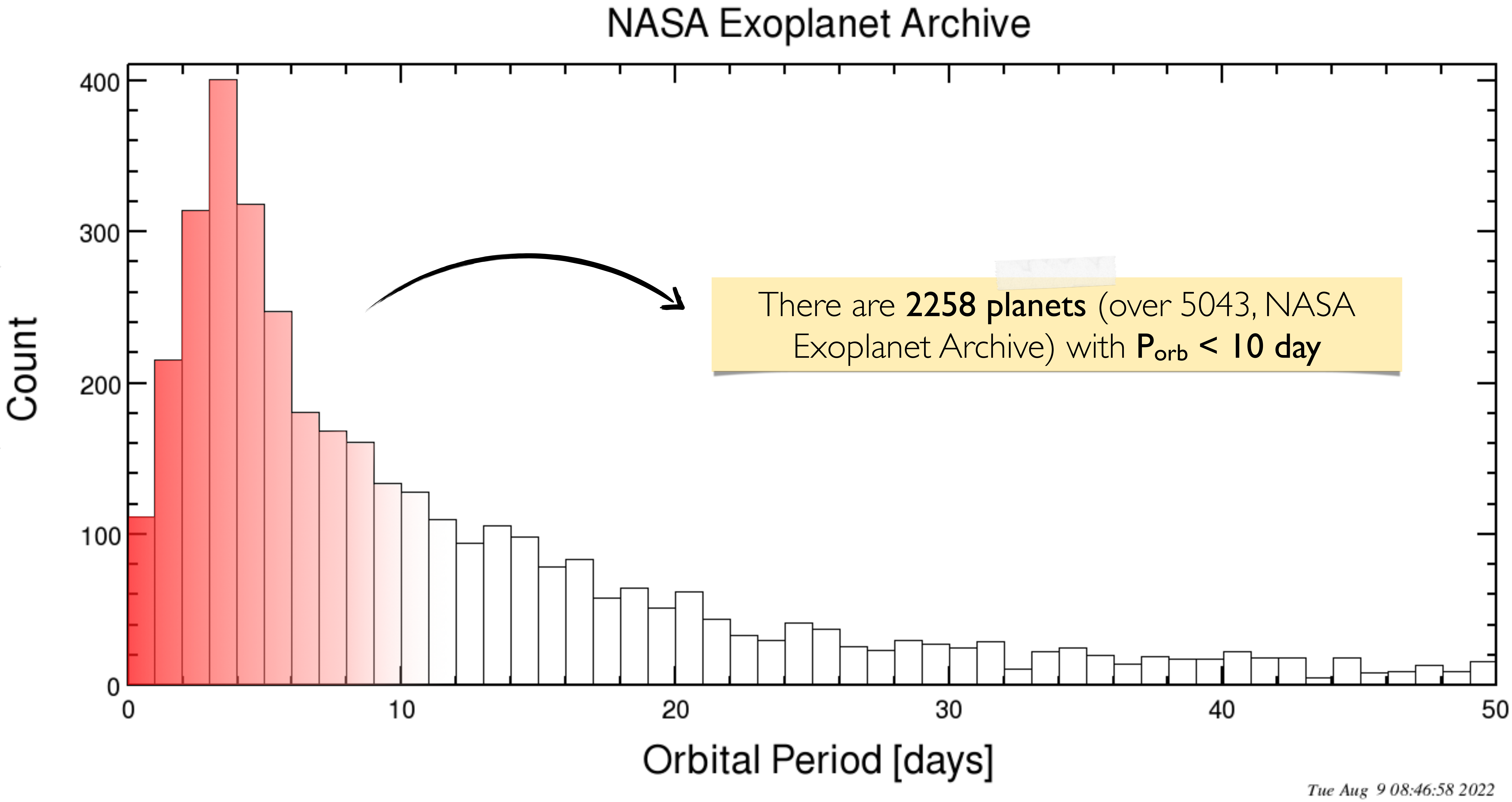
Star-planet tidal interactions

Tidal interactions

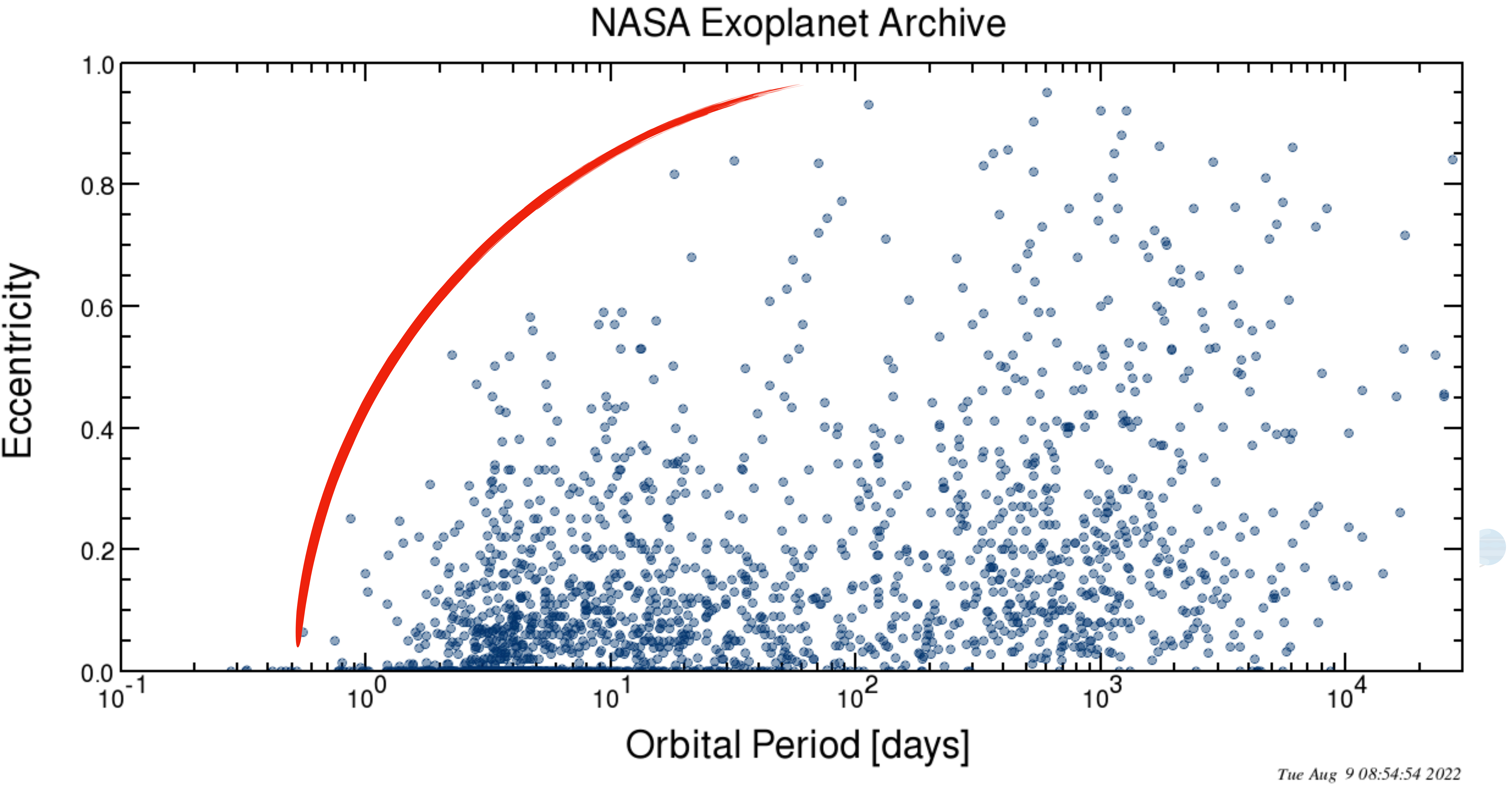
- Why are tides important?
- A bit of theory
  - ★ Tides, the easy way 😎
  - ★ Tides, the hard way… 🫣
- Some constraints brought by studying tides…
  - ★ Stars
  - ★ Planets
  - ★ Multi-planet systems

Tidal interactions

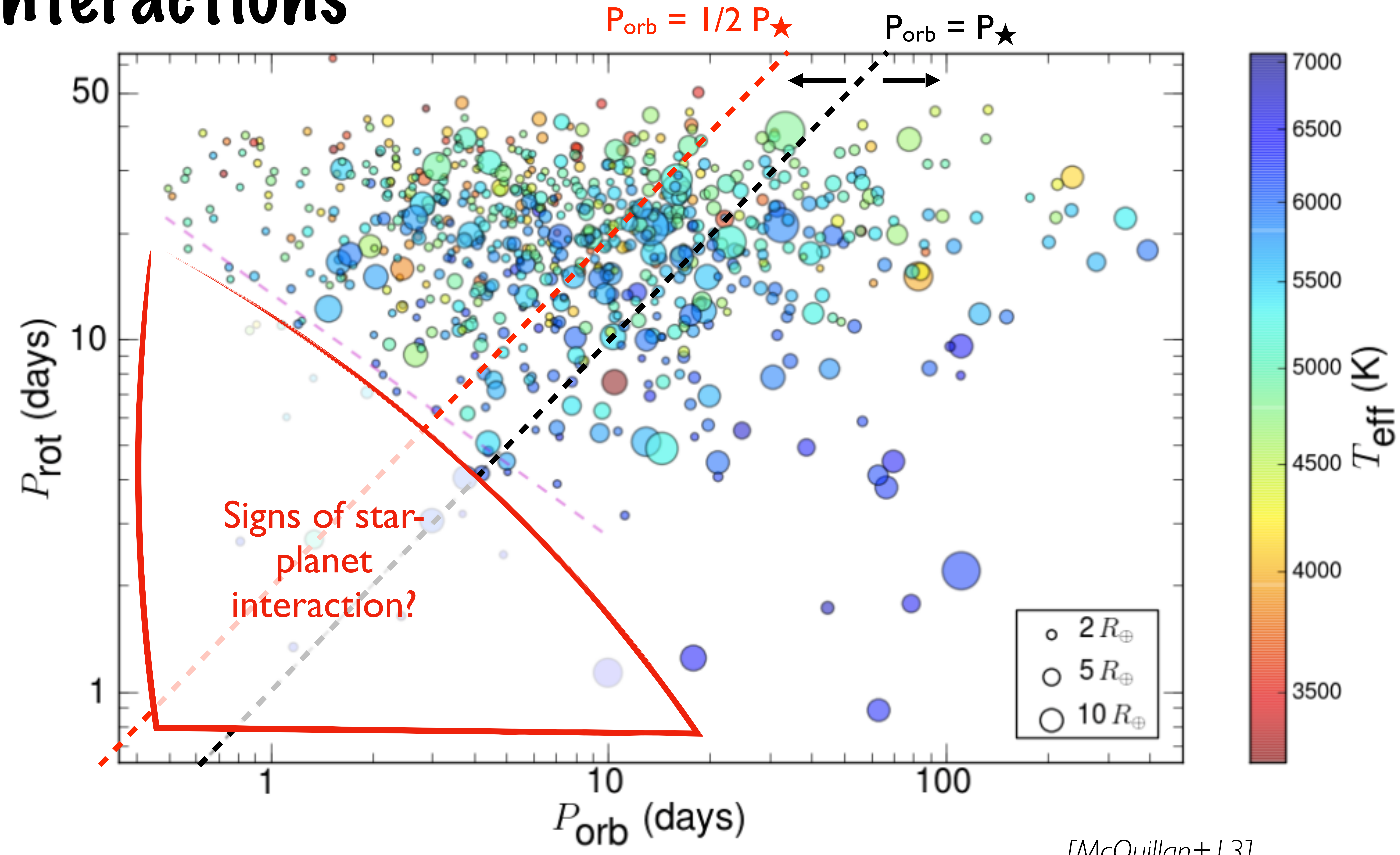


There are **2258 planets** (over 5043, NASA Exoplanet Archive) with **P_{orb} < 10 day**

Tidal interactions

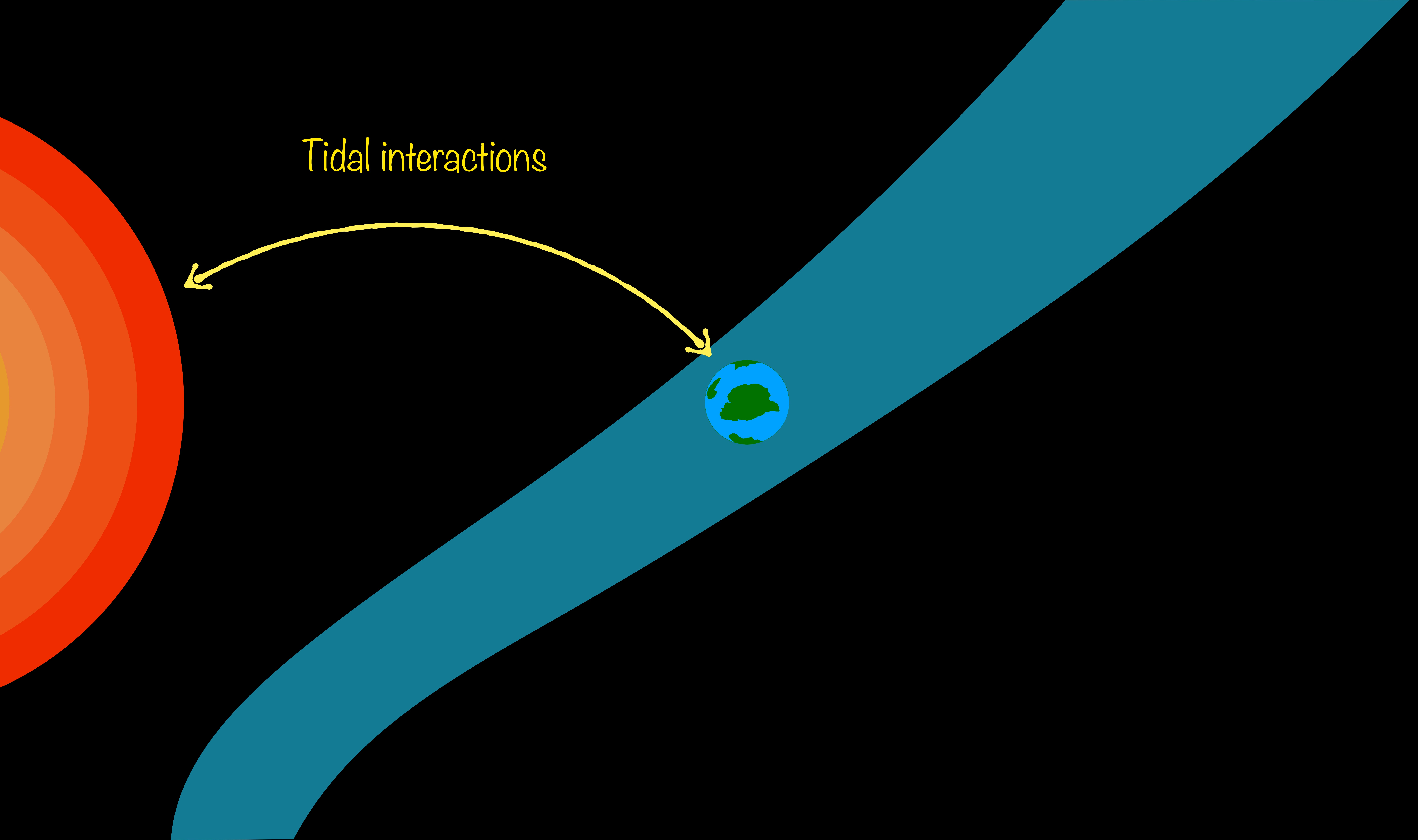


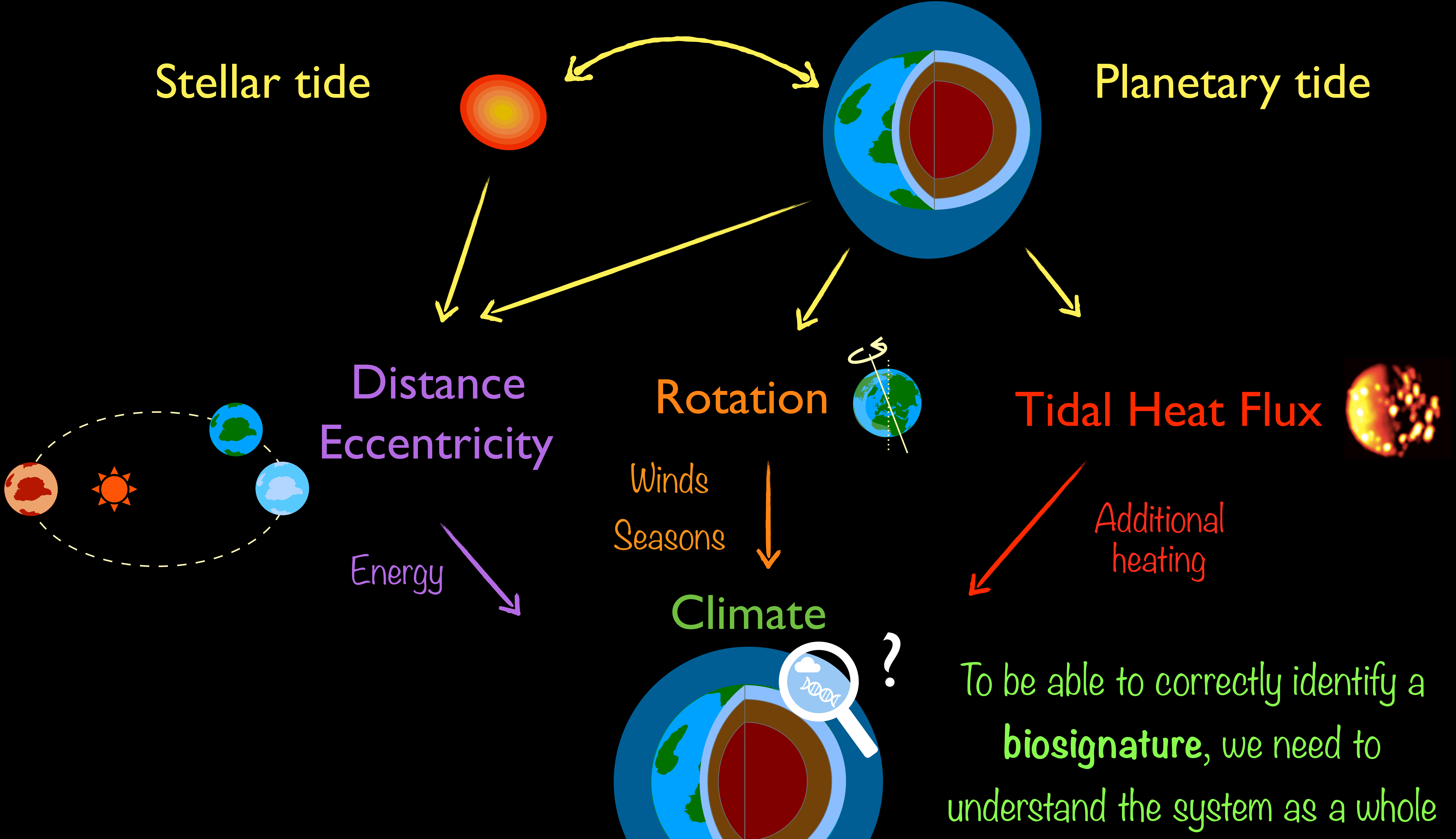
Tidal interactions



[McQuillan+13]

Solar system (not to scale)
Habitable Zone
Only Habitable Zone planets available for atmospheric characterization!
[e.g. Fauchez+18, Lovis+17]
System around cool dwarfs


Tidal interactions


Stellar tide
Planetary tide
Distance
Eccentricity
Rotation
Tidal Heat Flux
Energy
Winds
Seasons
Additional heating
Climate
?
To be able to correctly identify a **biosignature**, we need to understand the system as a whole

Tidal interactions

- Why are tides important?
- A bit of theory
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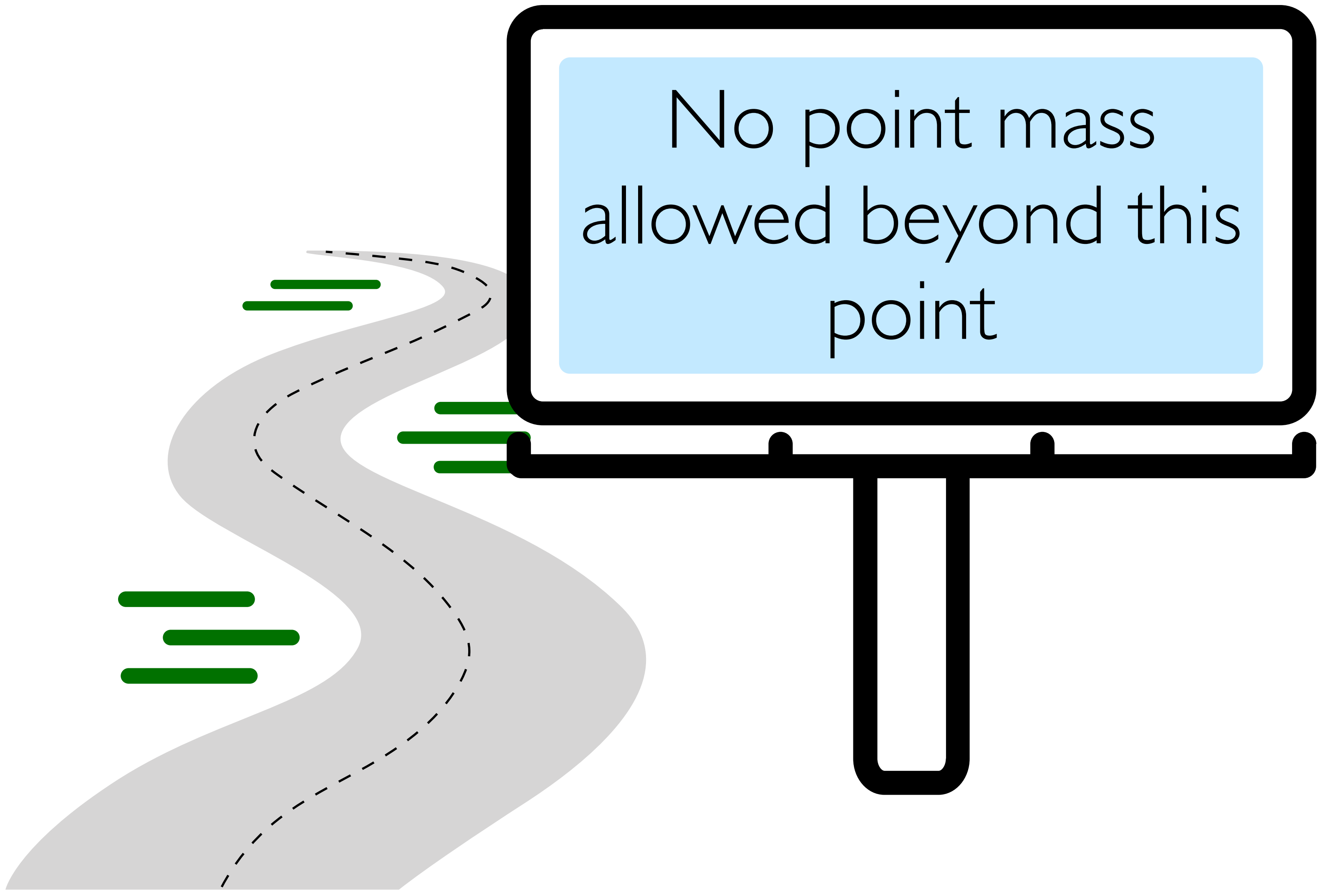
  - ★ Tides, the hard way… 🙈
- Some constraints brought by studying tides…
  - ★ Stars
  - ★ Planets
  - ★ Multi-planet systems

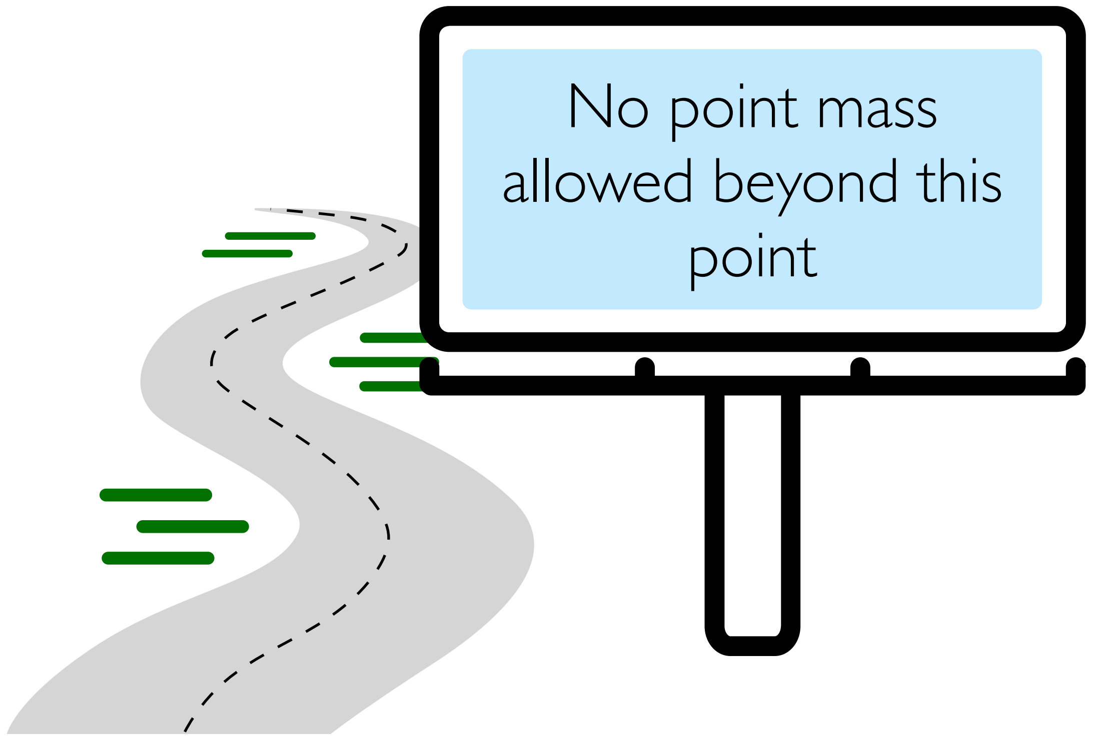
A bit of theory

Ingredients

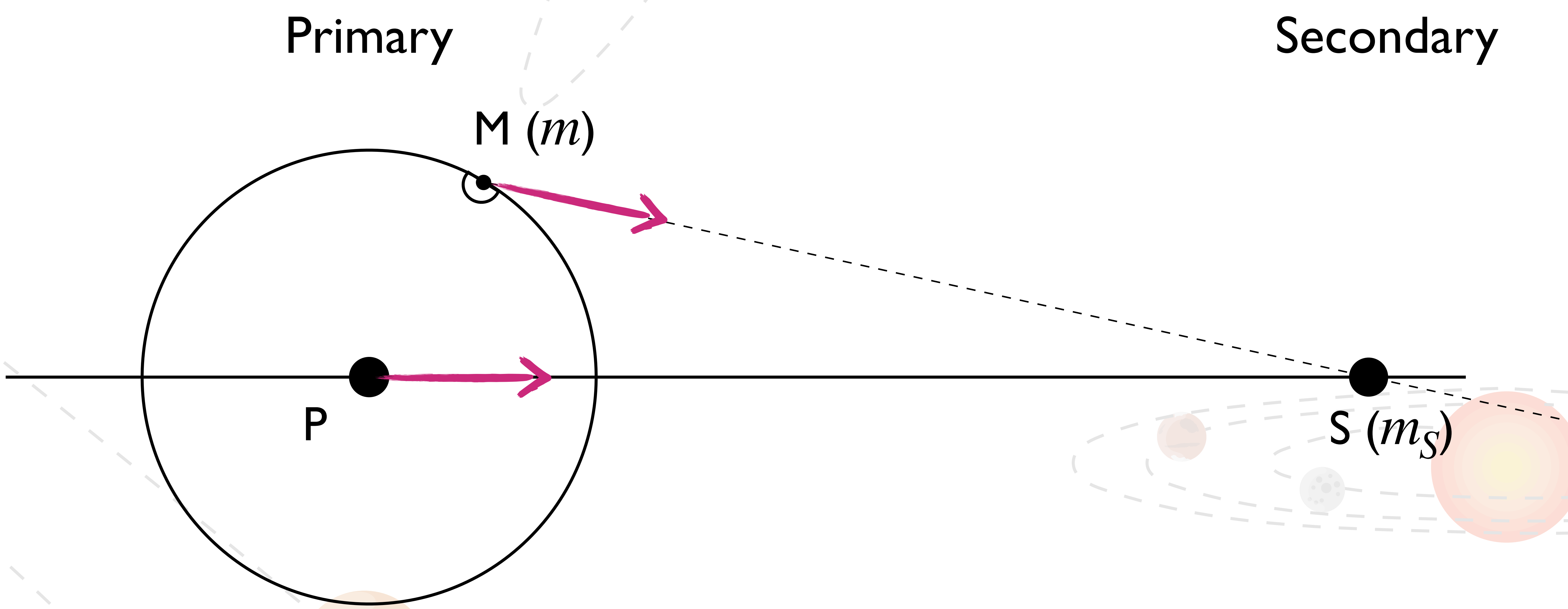
- **At least 2 objects** (could be star-planet, planet-satellite, star-star…)
- **Extended objects** (beyond newtonian point mass description)
- **Some kind of dissipative process** (e.g. thermal dissipation, viscous dissipation…)
- **Objects shouldn't be too far from each other**
- **Some time**



Let's start simple



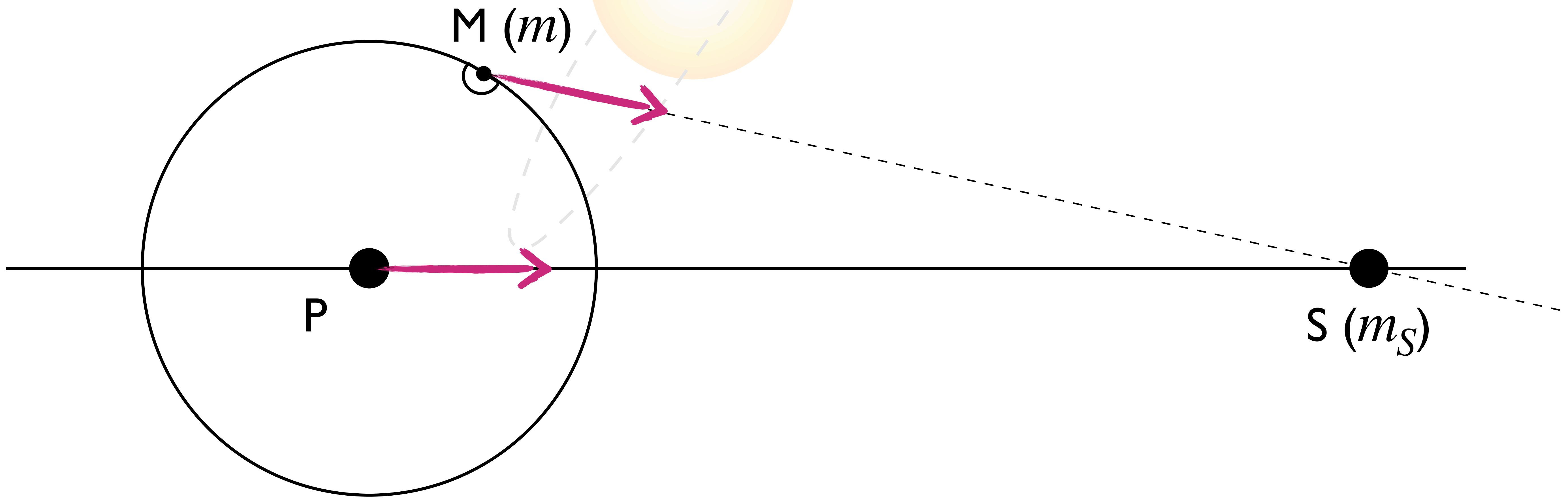
Tidal interaction appears when we consider that a **body is not a point mass** but **extended**



Tides are a **differential gravitational effect**

Tidal field

Tidal interaction appears when we consider that a **body is not a point mass** but **extended**



Acceleration (in primary frame, which is accelerating at $\mathbf{a}_P = \mathbf{G}(P)$):

$$m\mathbf{a}_M|_P = m\mathbf{G}(M) + \mathbf{F}' - m\mathbf{G}(P)$$

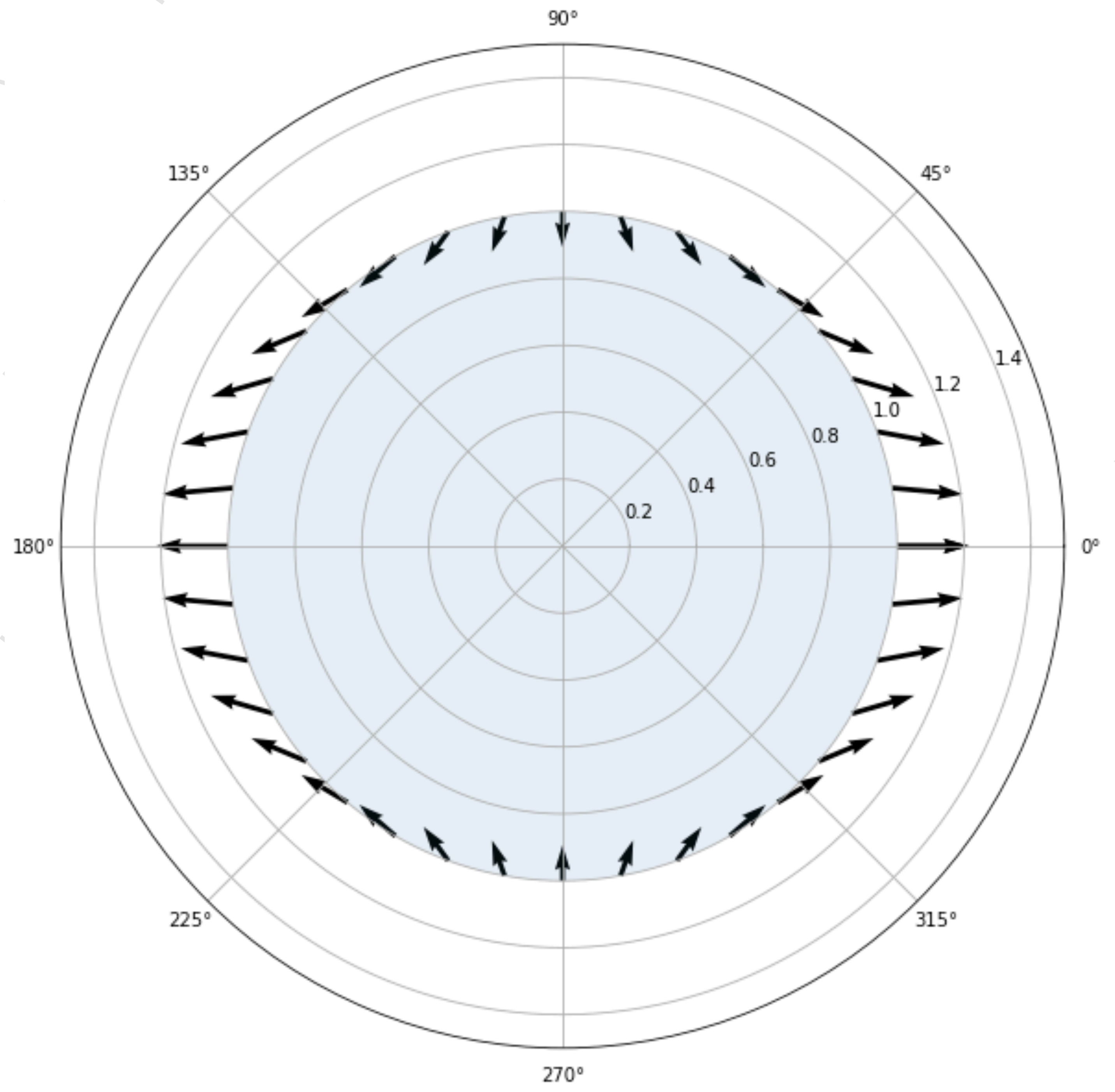
$$m\mathbf{a}_M|_P = m\underbrace{(\mathbf{G}(M) - \mathbf{G}(P))}_{\text{Tidal field } \mathbf{C}(M)} + \mathbf{F}'$$

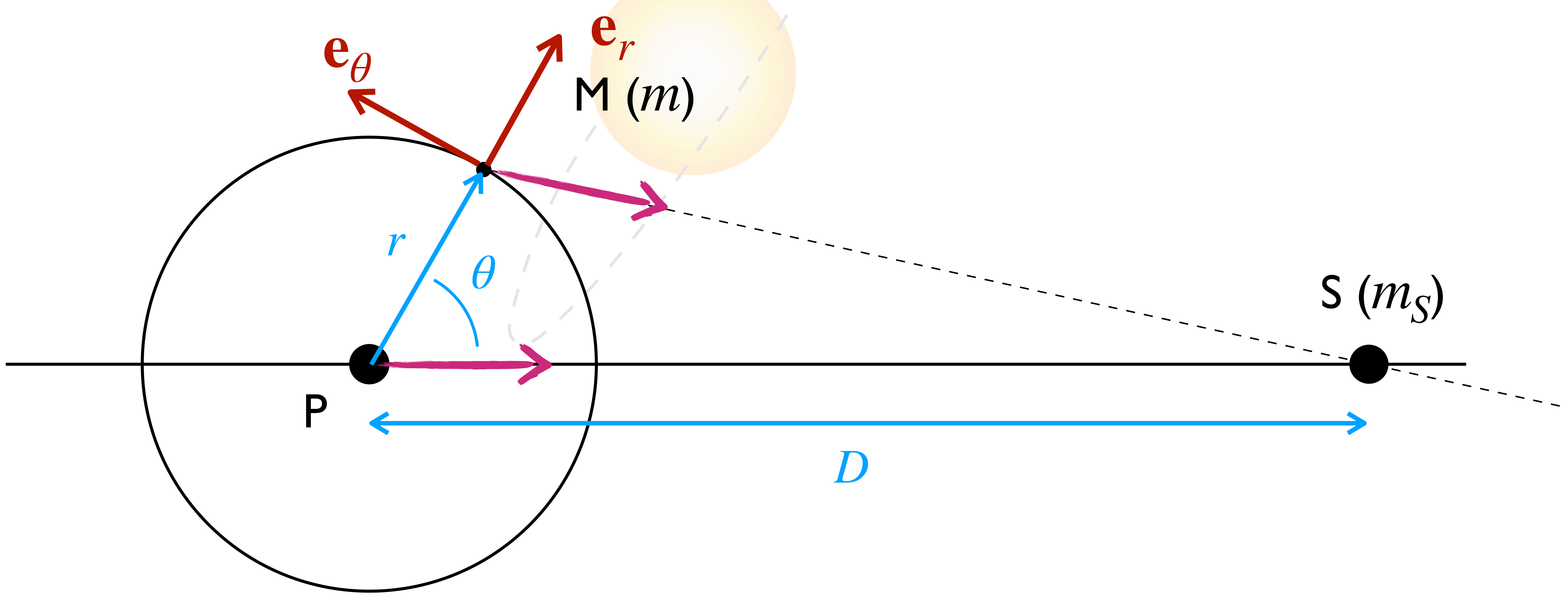
Note that the primary frame here is not rotating, other terms would appear if we were to consider a co-rotating frame (rotation + Coriolis)

The tidal field $\mathbf{C}(M) = \mathbf{G}(M) - \mathbf{G}(P)$ appears only when passing from an inertial frame to the primary-centered frame

Tidal field

Tidal interaction appears when we consider that a **body is not a point mass** but **extended**





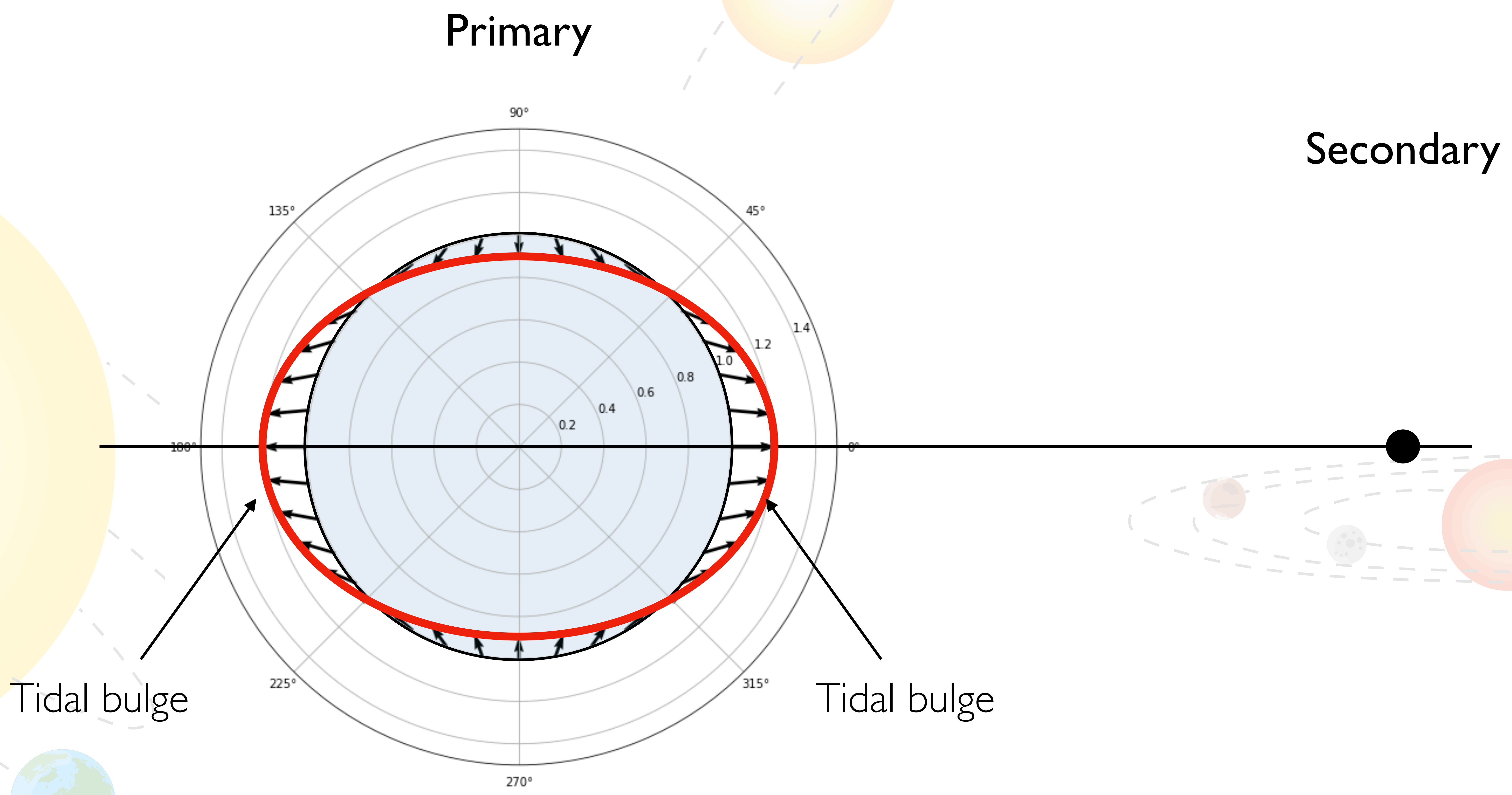
The tidal field is given by:

$$\mathbf{C}(M) = Gm_S \frac{r}{D^3}\left(C_r \mathbf{e}_r + C_\theta \mathbf{e}_\theta\right) \text{ where}$$

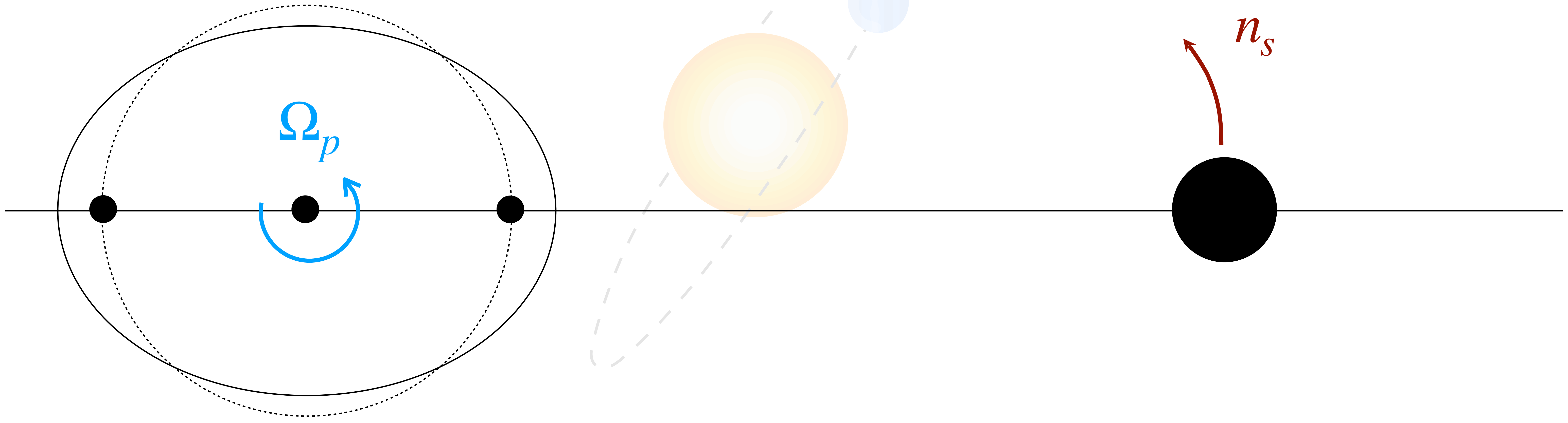
$$C_r = 3\cos^2\theta - 1 \text{ and } C_\theta = -\frac{3}{2}\sin 2\theta$$

Tidal field

Tidal interaction appears when we consider that a **body is not a point mass** but **extended**



Simple case: coplanar, circular



Rotation frequency of primary

Ω_p

Orbital frequency of secondary $\propto 1/P$

n_s

Three different configurations:

★ $\Omega_p = n_s$

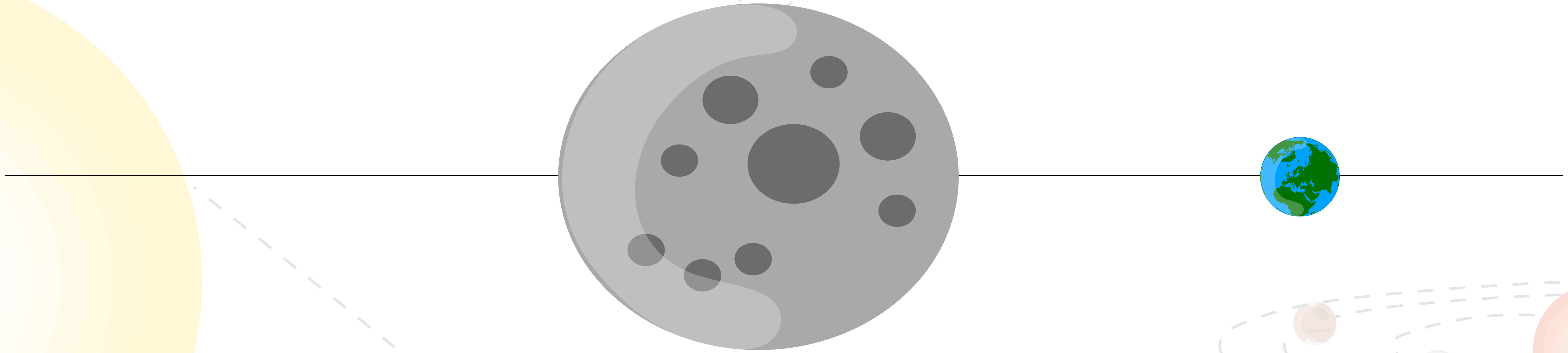
★ $\Omega_p < n_s$

★ $\Omega_p > n_s$

Simple case: coplanar, circular

★ $\Omega_p = n_s$

Orbital distance of secondary $a_s = r_c$ Corotation radius



Earth-Moon system

It's the **tidal equilibrium state!**

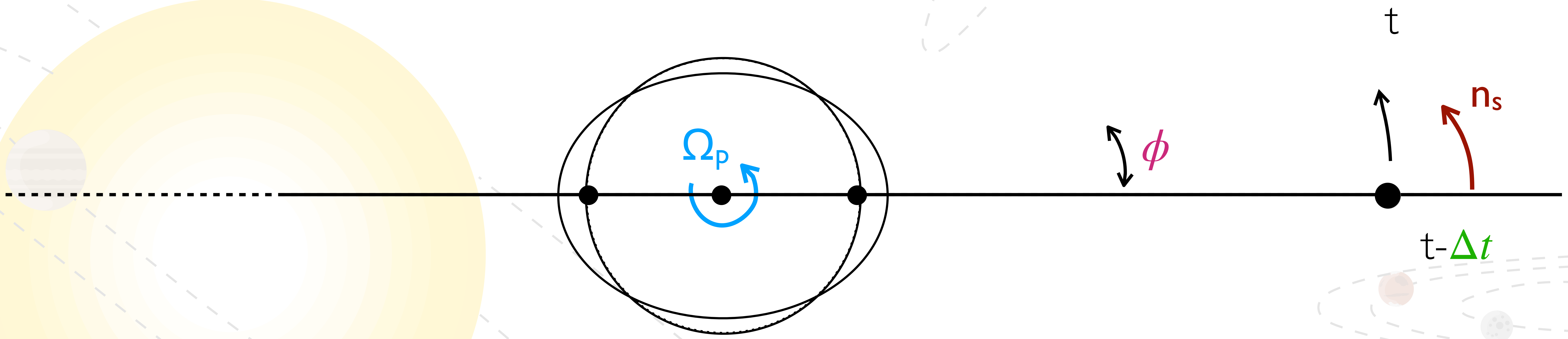
Simple case: coplanar, circular

★ $\Omega_p < n_s$

$a_s < r_c$

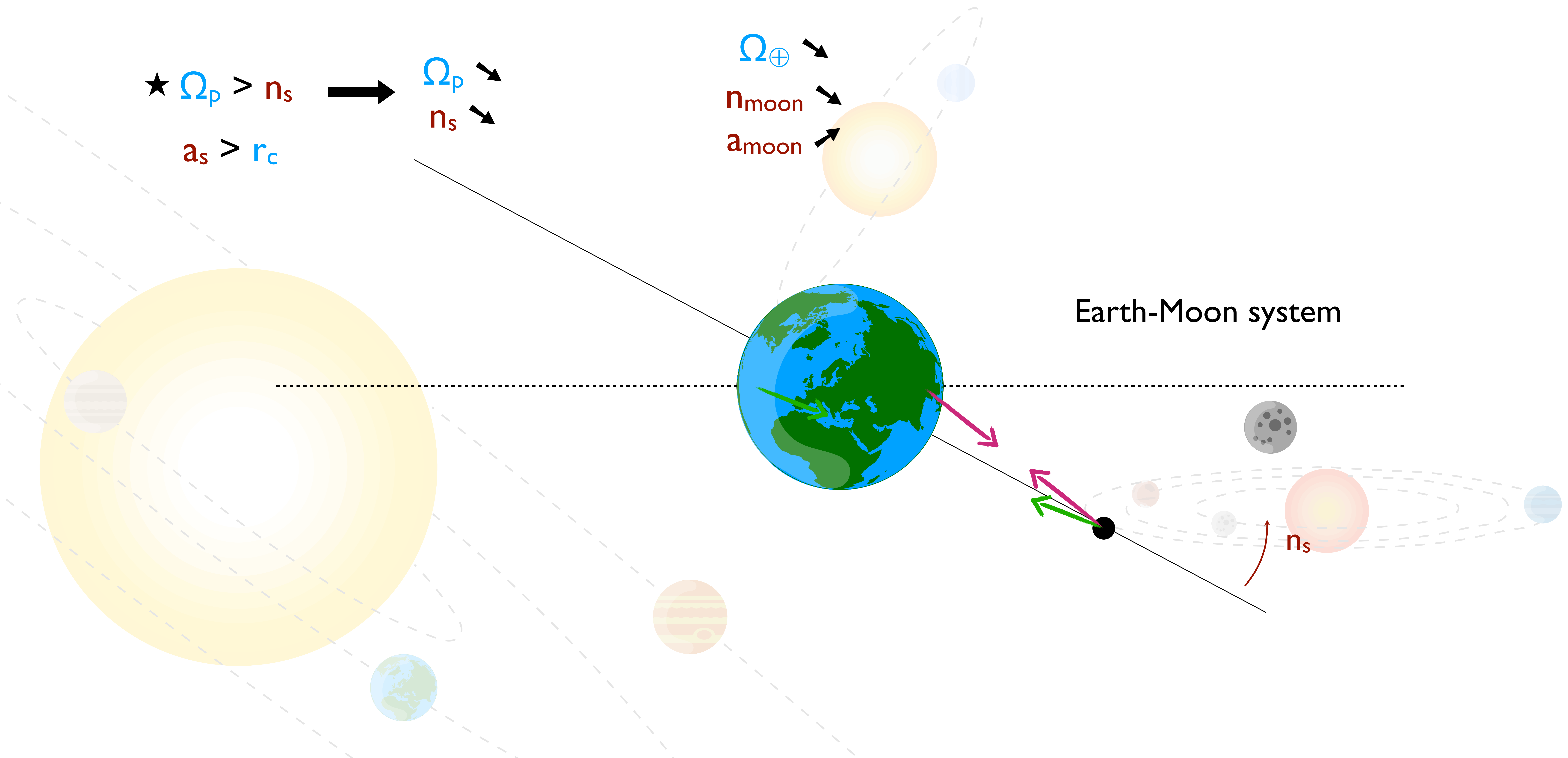
Constant **time** lag model (Δt): e. g. *Mignard, 1979; Hut 1981*

Constant **phase** lag model (Q or Q'): e. g. *Goldreich 1963*



Simple case: coplanar, circular

Simple case: coplanar, circular

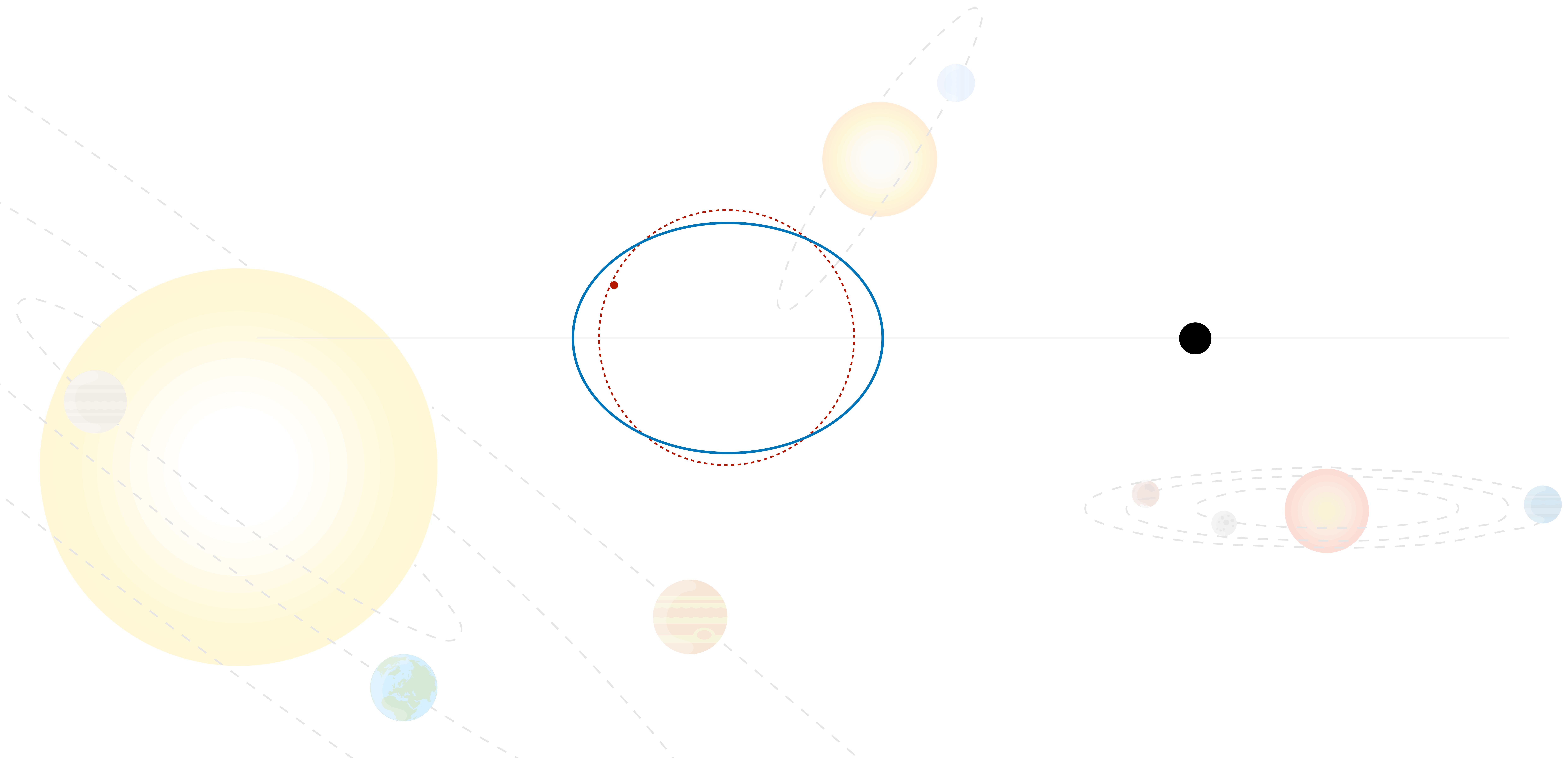


Simple case: coplanar, circular

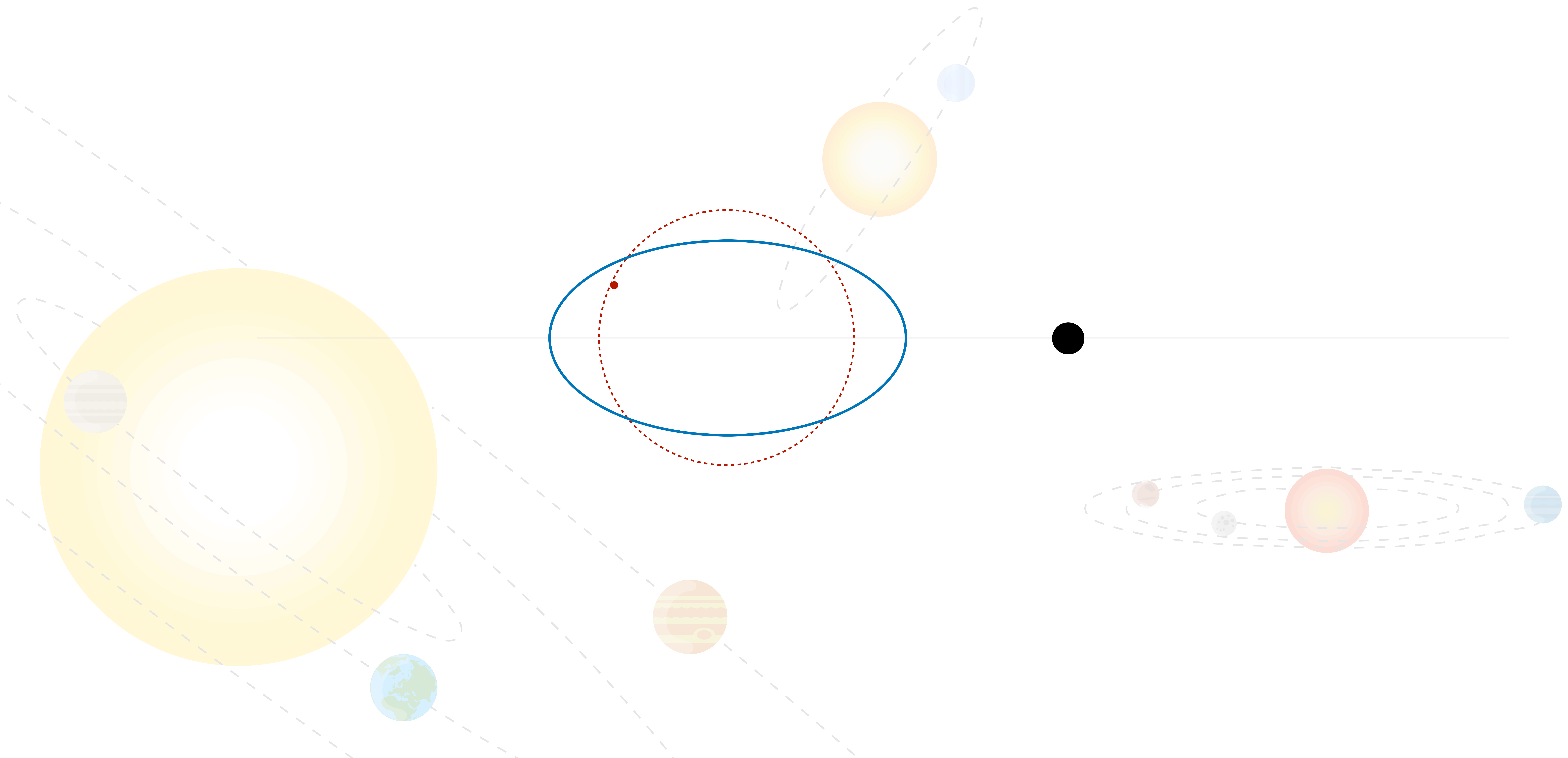
Minimize the deformation/dissipation

- Synchronize the rotation

Not so simple case: eccentricity and obliquity



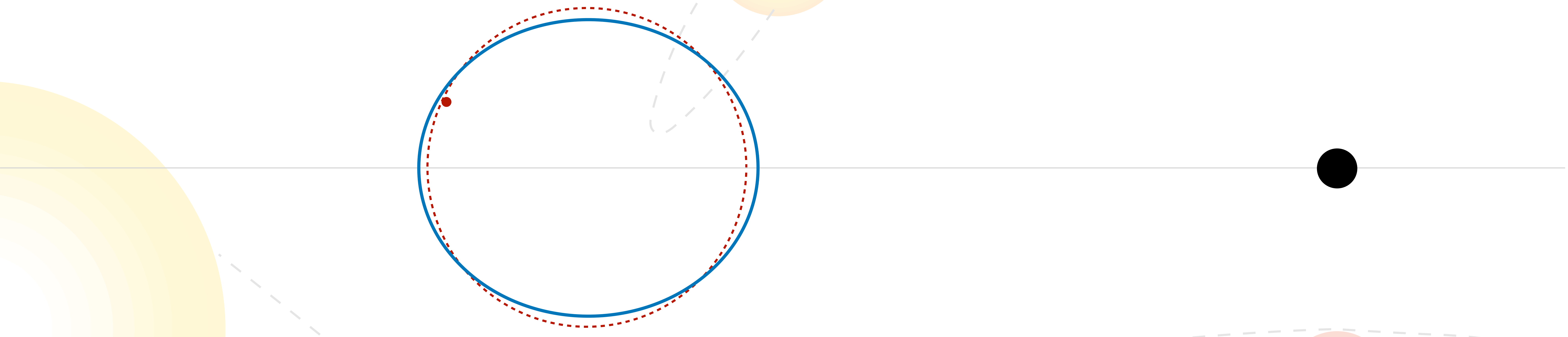
Not so simple case: eccentricity and obliquity



Not so simple case: eccentricity and obliquity

Minimize the deformation/dissipation

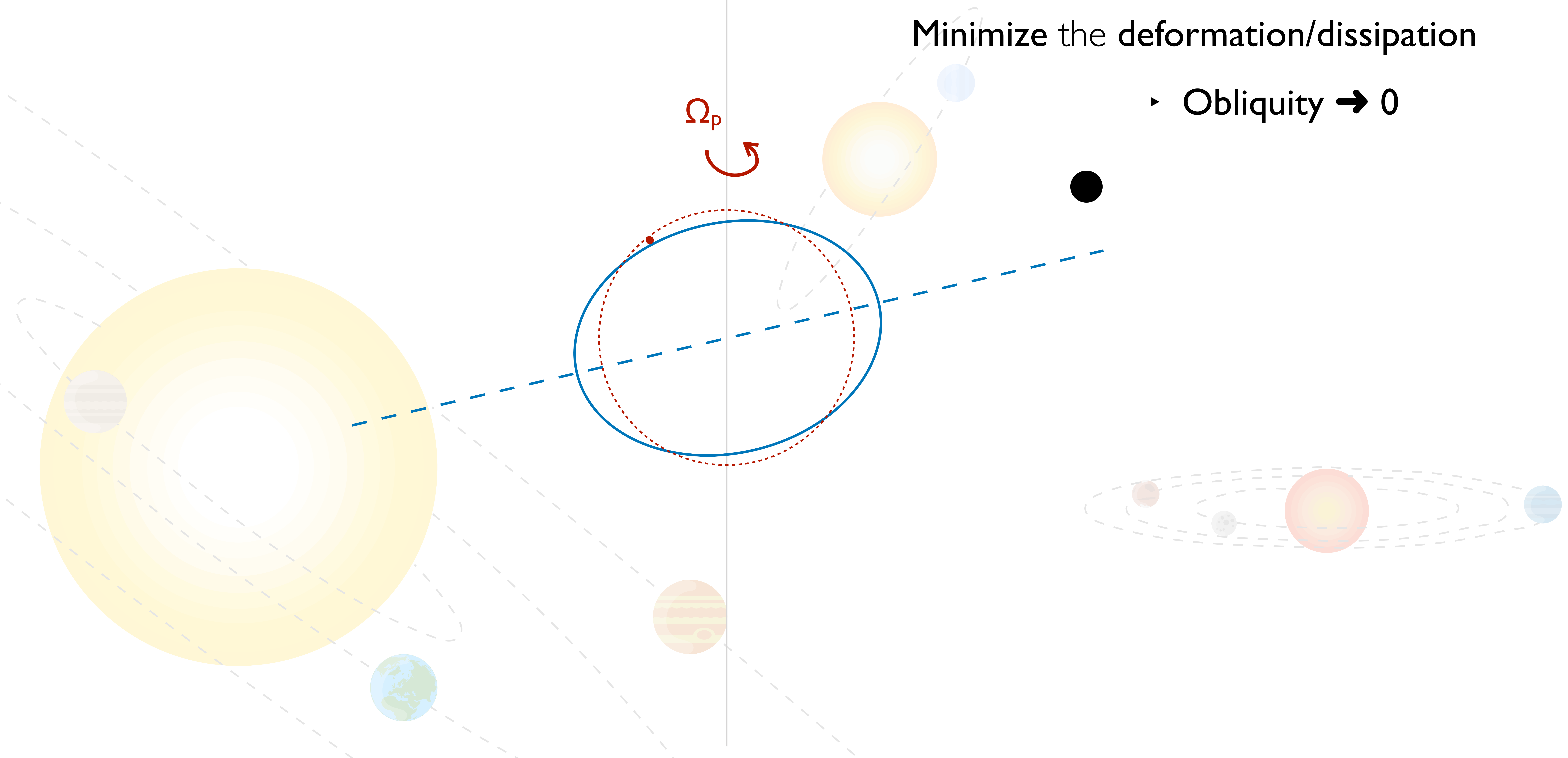
- Eccentricity ➜ 0



Not so simple case: eccentricity and obliquity

Minimize the deformation/dissipation

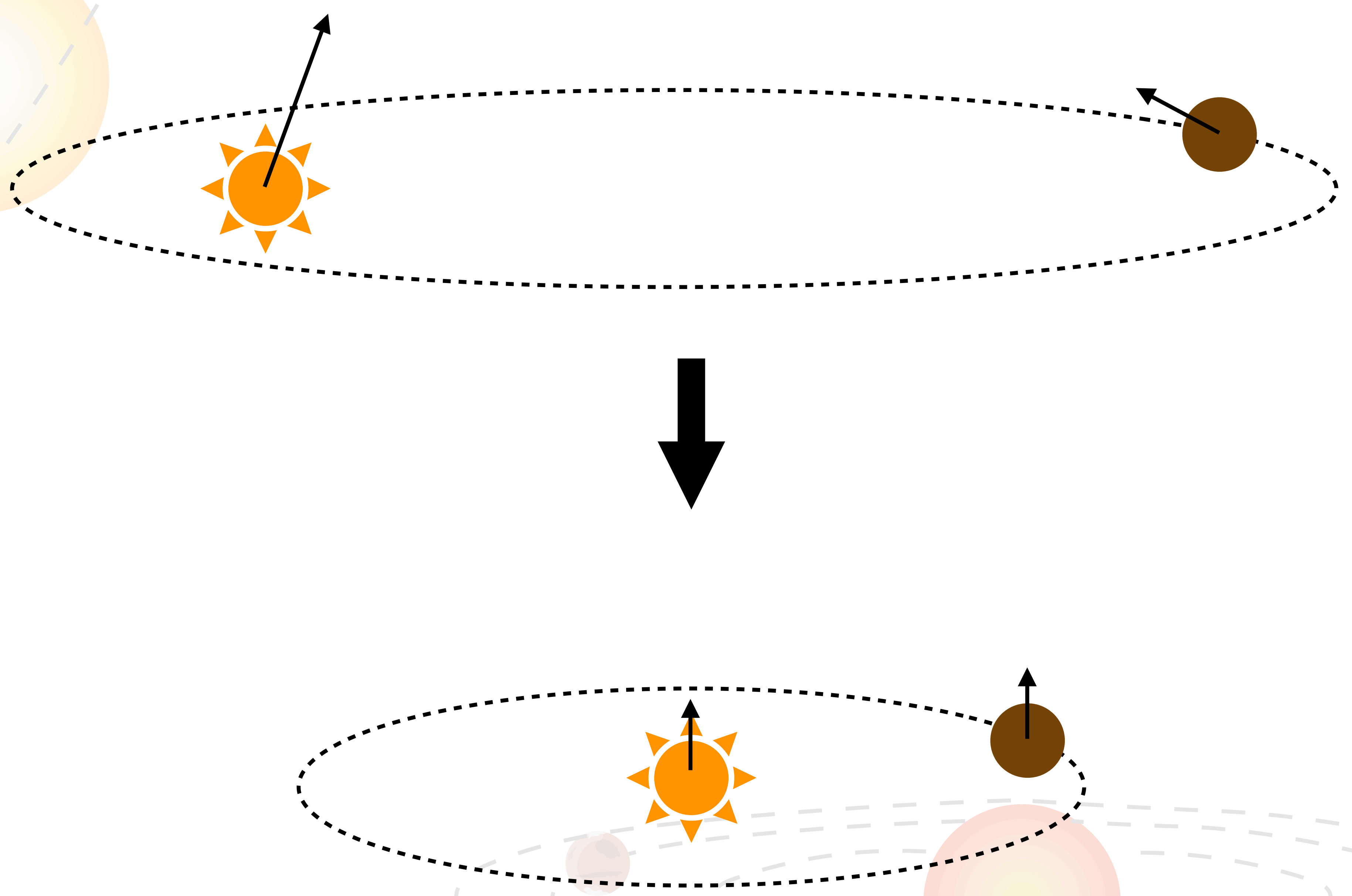
- Obliquity ➜ 0



Not so simple case: rotation, eccentricity and obliquity

Minimize the deformation/dissipation

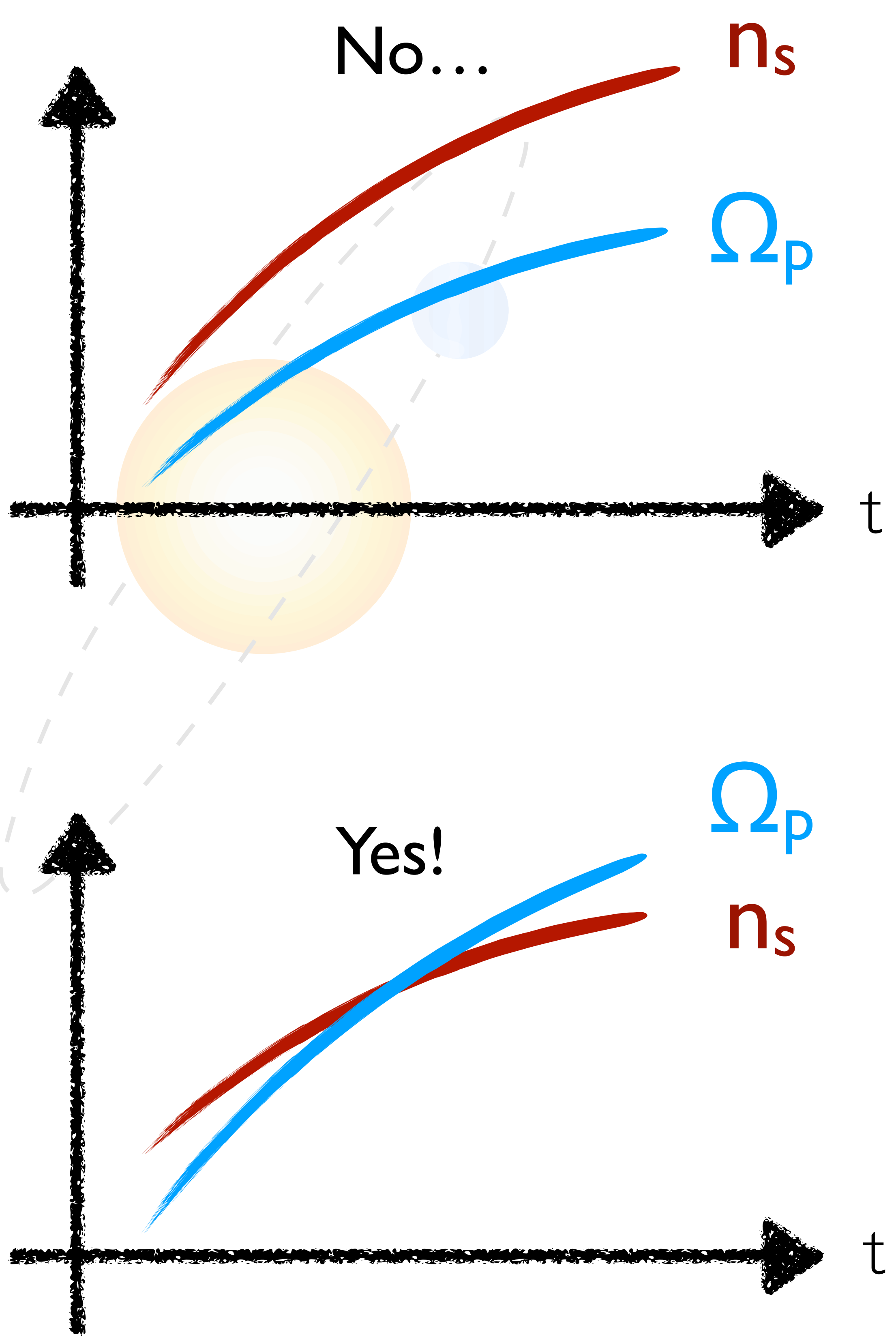
- Eccentricity ➜ 0
- Obliquity ➜ 0
- Synchronize the rotation



$$\Omega_\star = \Omega_p = n$$

Is an equilibrium always possible?

- ★ $\Omega_p < n_s$ ⟶ Ω_p ↗, n_s ↗
- ★ $\Omega_p > n_s$ ⟶ Ω_p ↘, n_s ↘



Can the system reach an **equilibrium** ($\Omega_p = n_s$)?

Is an equilibrium always possible?

★ $\Omega_p < n_s$ ⟶ Ω_p ↗ n_s ↗

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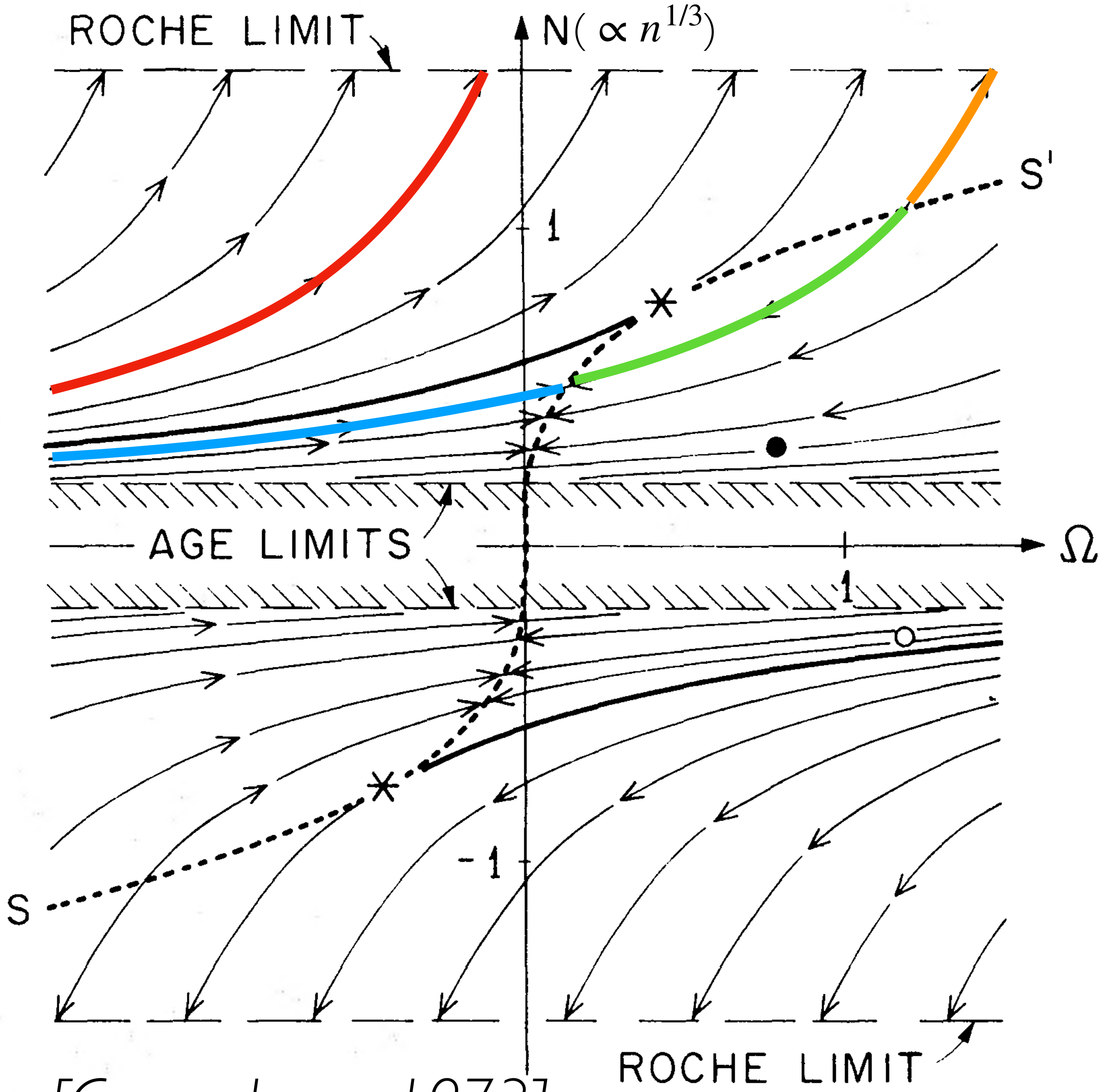
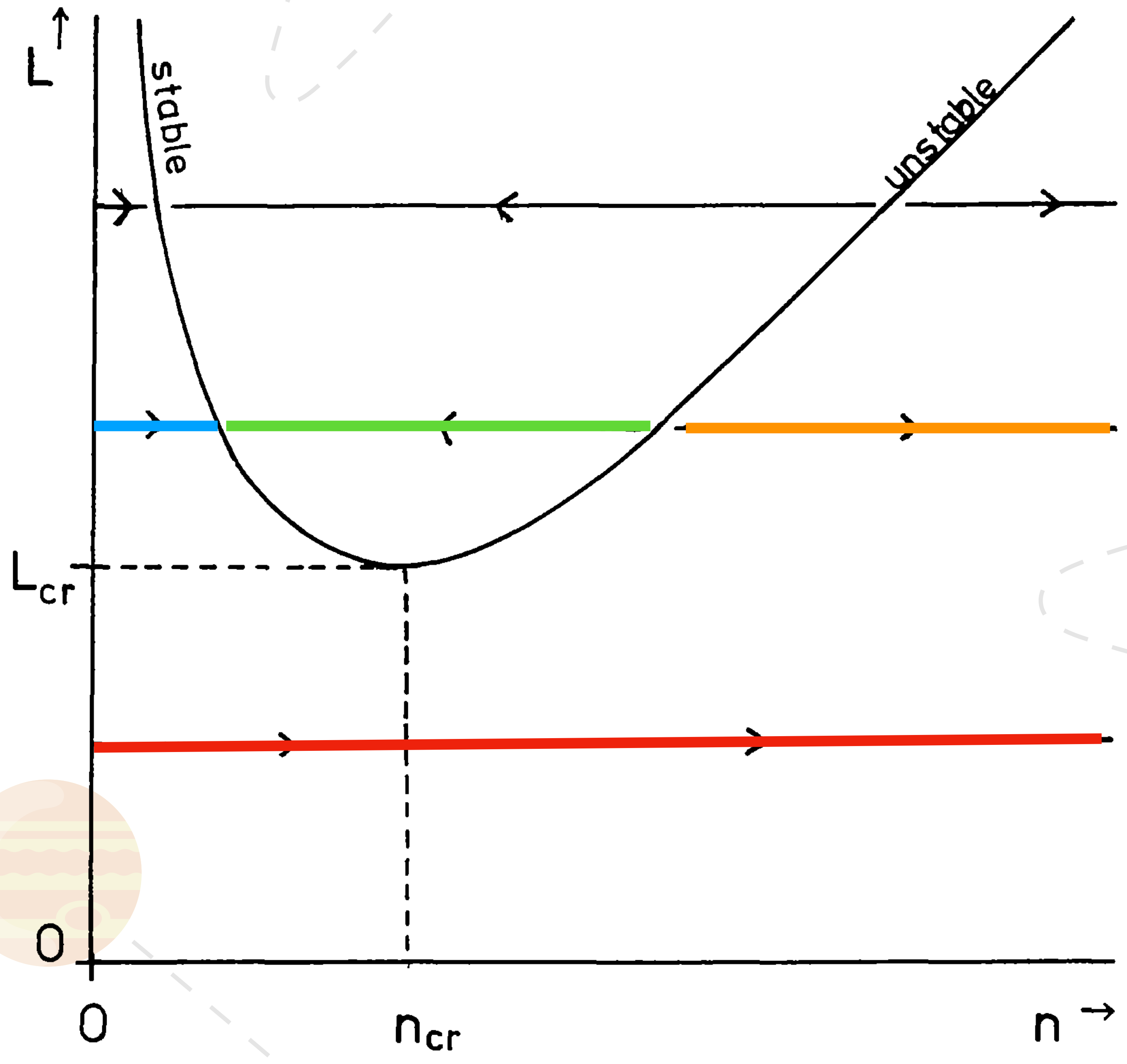
Can the system reach an **equilibrium** ($\Omega_p = n_s$)?



[Counselman, 1973]



[Hut 1980]

L is the **total angular momentum** (conserved quantity)
h is the orbital angular momentum

Equilibrium exists if **$L > L_{crit}$** (which depends on the masses and moments of inertia)

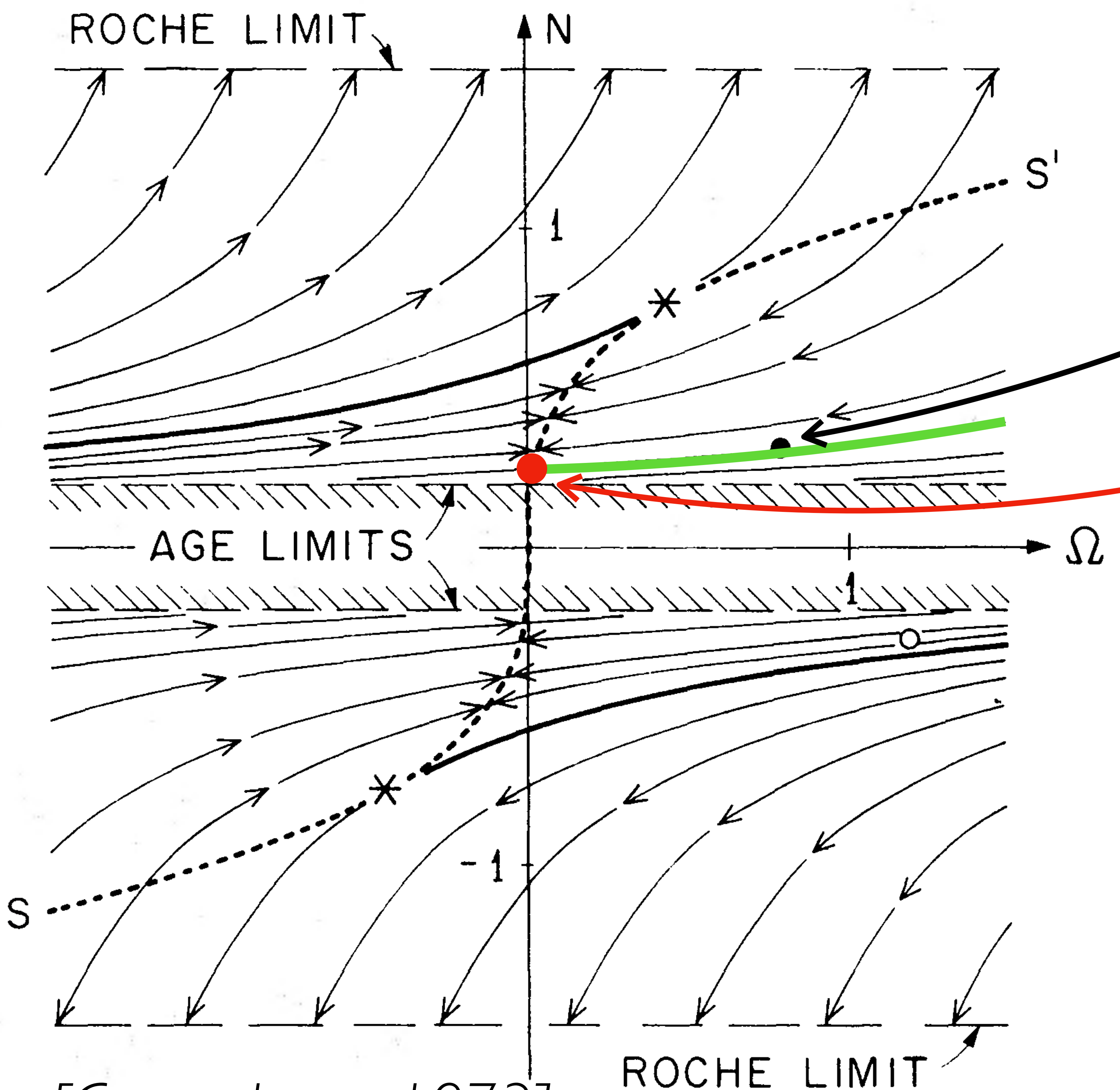
Stable equilibrium reachable if **h > 3/4 L**

Is an equilibrium always possible?

★ $\Omega_p < n_s$ ⟶ Ω_p ↗ n_s ↗

★ $\Omega_p > n_s$ ⟶ Ω_p ↘ n_s ↘

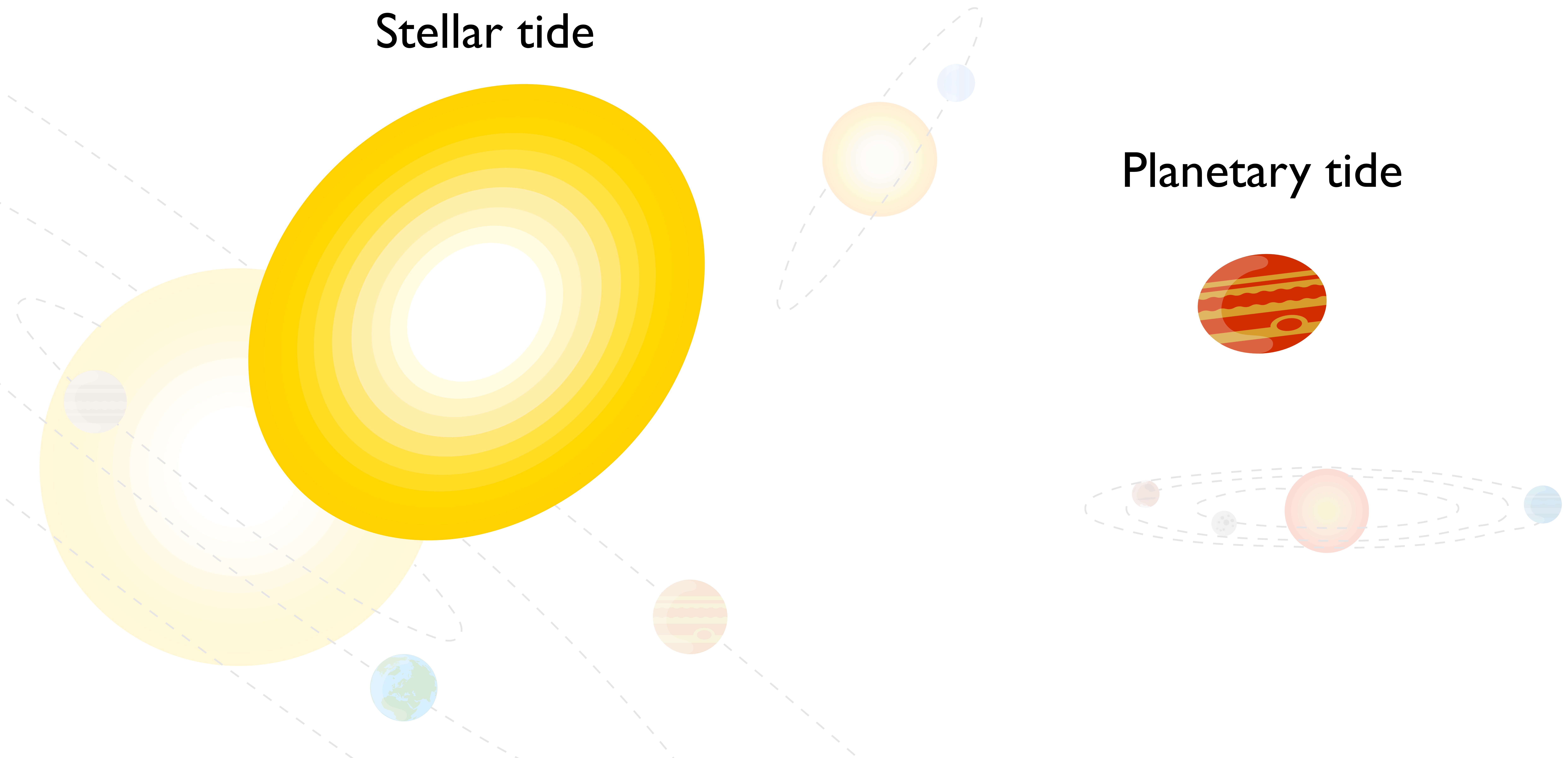
Can the system reach an **equilibrium** ($\Omega_p = n_s$)?



[Counselman, 1973]

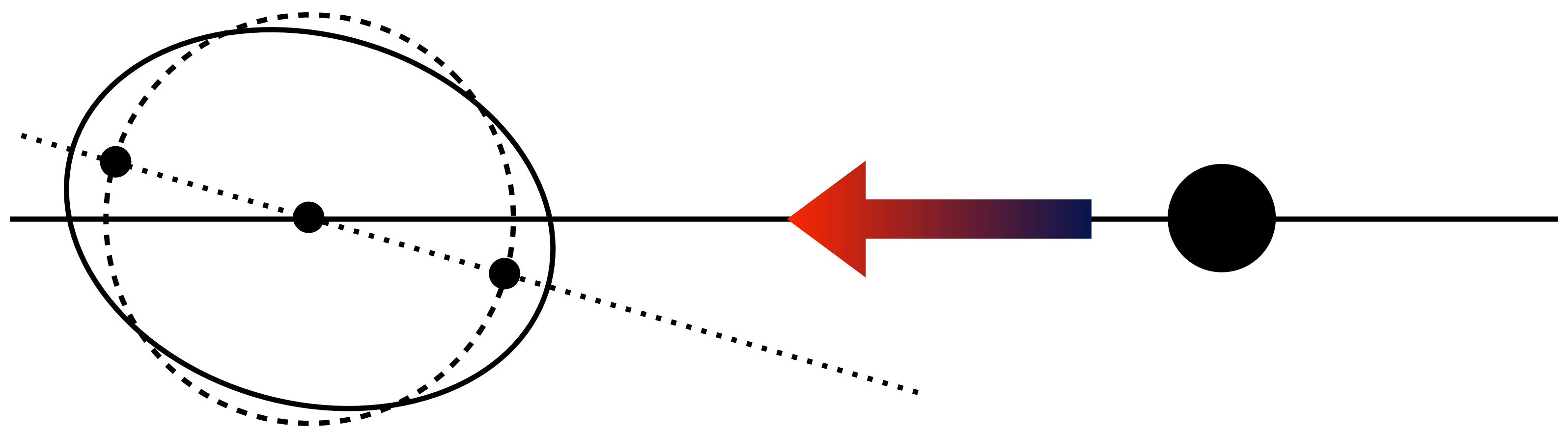
Earth-Moon system today ($a = 60\ R_\oplus$, $P_\oplus = 24$ day)

Equilibrium at $a \approx 90\ R_\oplus$ and $P_\oplus \approx 52$ day

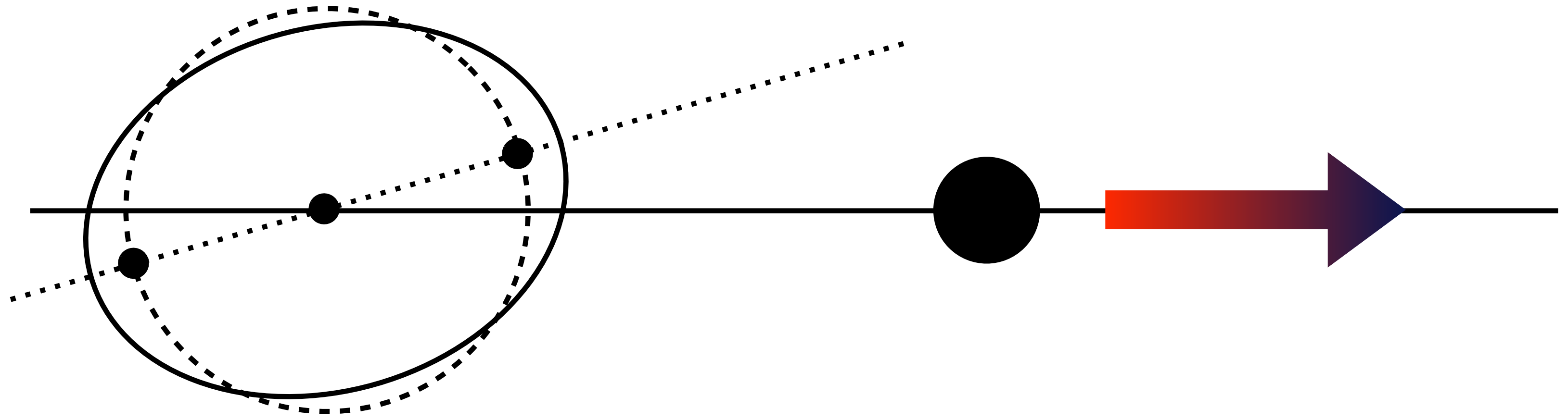

Stellar tide
Planetary tide

Stellar tide

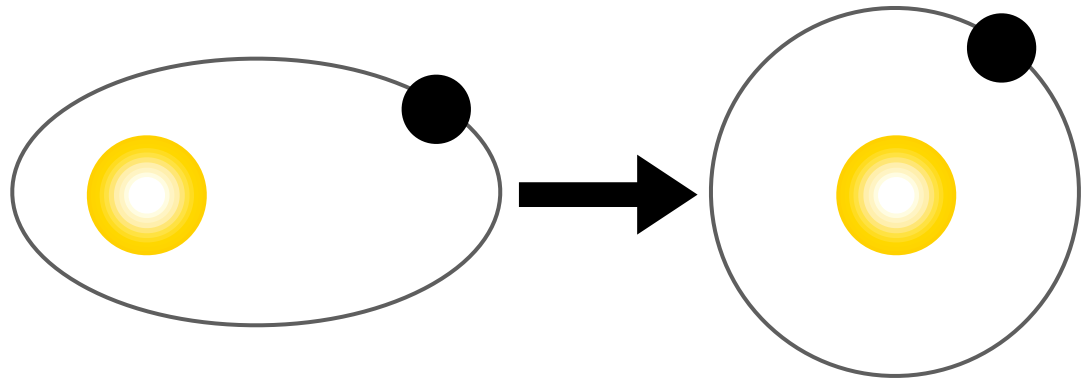
- Planet **inside corotation** ➜ planet **migrates inward**



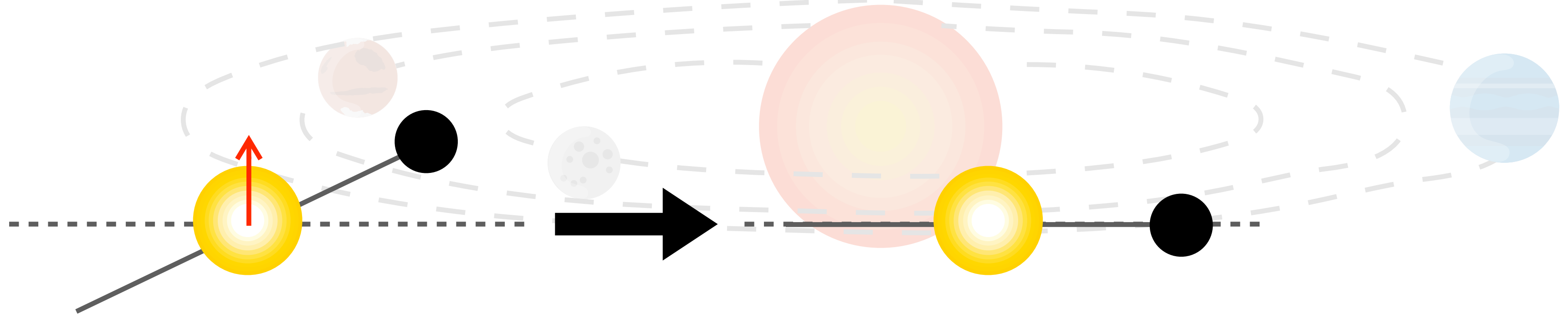
- Planet **outside corotation** ➜ planet **migrates outward**



- Eccentricity **decreases**

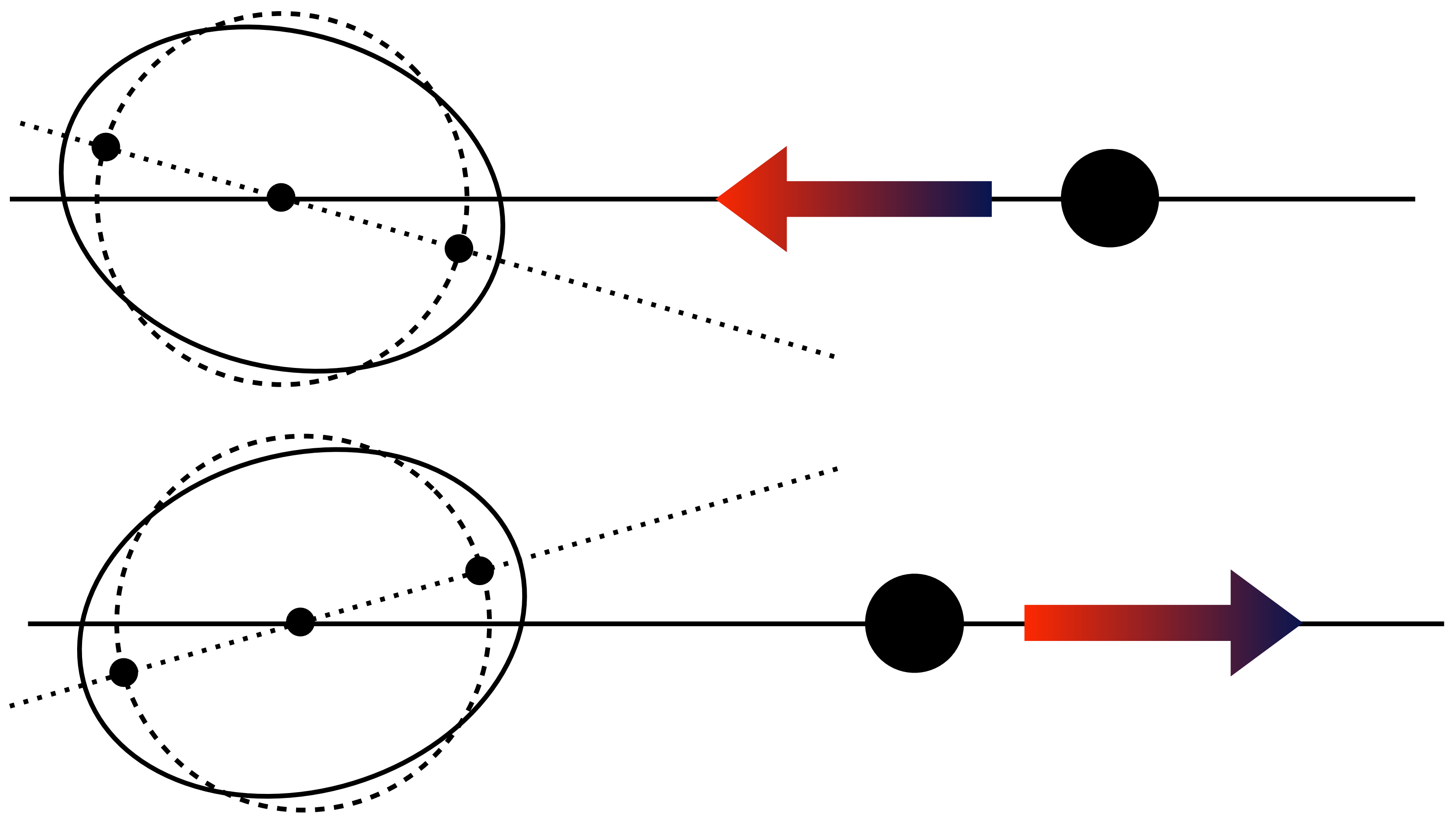


- Inclination of planetary orbit **decreases**



- **Timescales** depend on **stellar radius** and the **stellar dissipation**

Stellar tide



- Planet inside corotation ➜ planet migrates inward
- Planet outside corotation ➜ planet migrates outward
- Eccentricity **decreases**
- Inclination of planetary orbit **decreases**
- **Timescales** depend on **stellar radius** and the **stellar dissipation**

In many articles, you might find the **tidal quality factor** Q (or **time lag** Δt)

Low Q (high Δt) means a **fast evolution**

High Q (low Δt) means a **slow evolution**

Stars: $Q_\star \approx 10^5 - 10^8$ *[Penev, 2018]*

Planetary tide

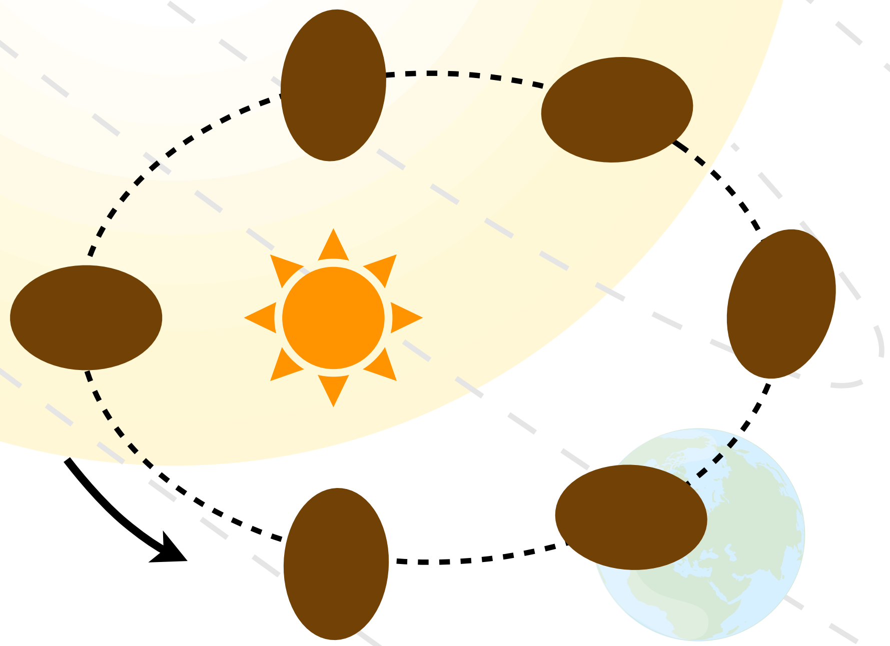
- **Circular** orbit: quick synchronization
- **Eccentric** orbit: quick pseudo-synchronization / spin-orbit resonance

Weakly viscous fluid approximation

e.g. constant time lag model *[e.g. Hut 1981]*

Eccentricity = 0 ➜ Synchronization

Eccentricity ≠ 0 ➜ Pseudo-synchronization

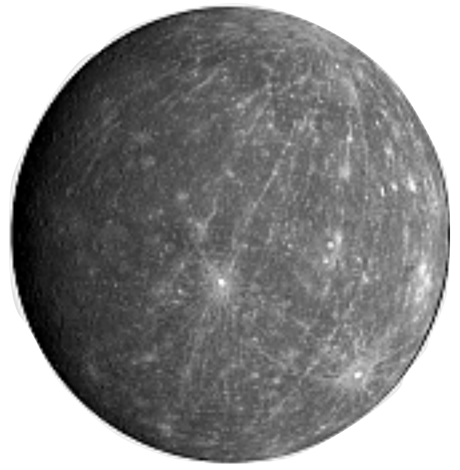


Anelastic material approximation

e.g. Andrade rheology *[e.g. Efroimsky+, Makarov+13]*

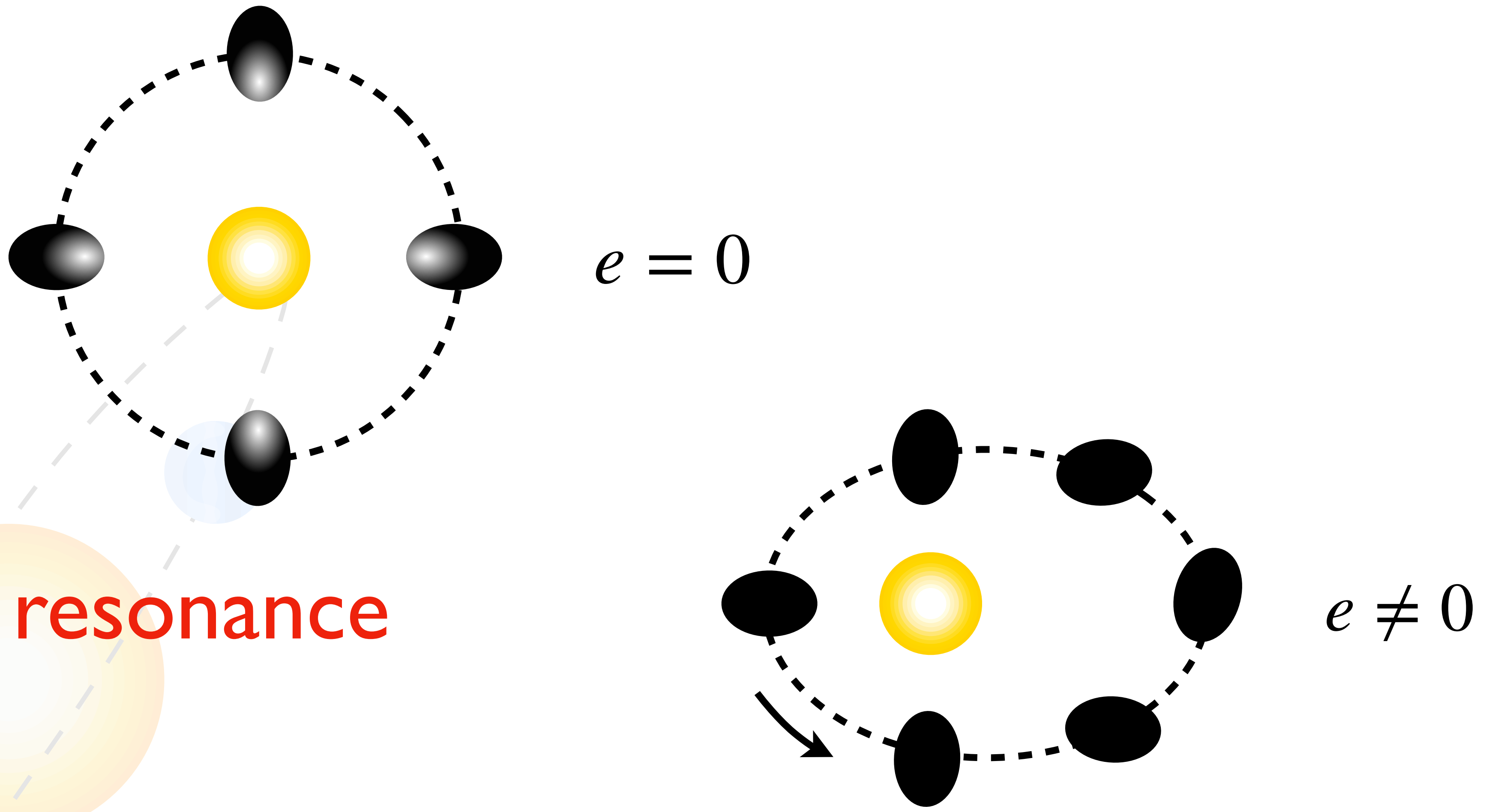
Eccentricity = 0 ➜ Synchronization

Eccentricity ≠ 0 ➜ Spin-orbit resonance

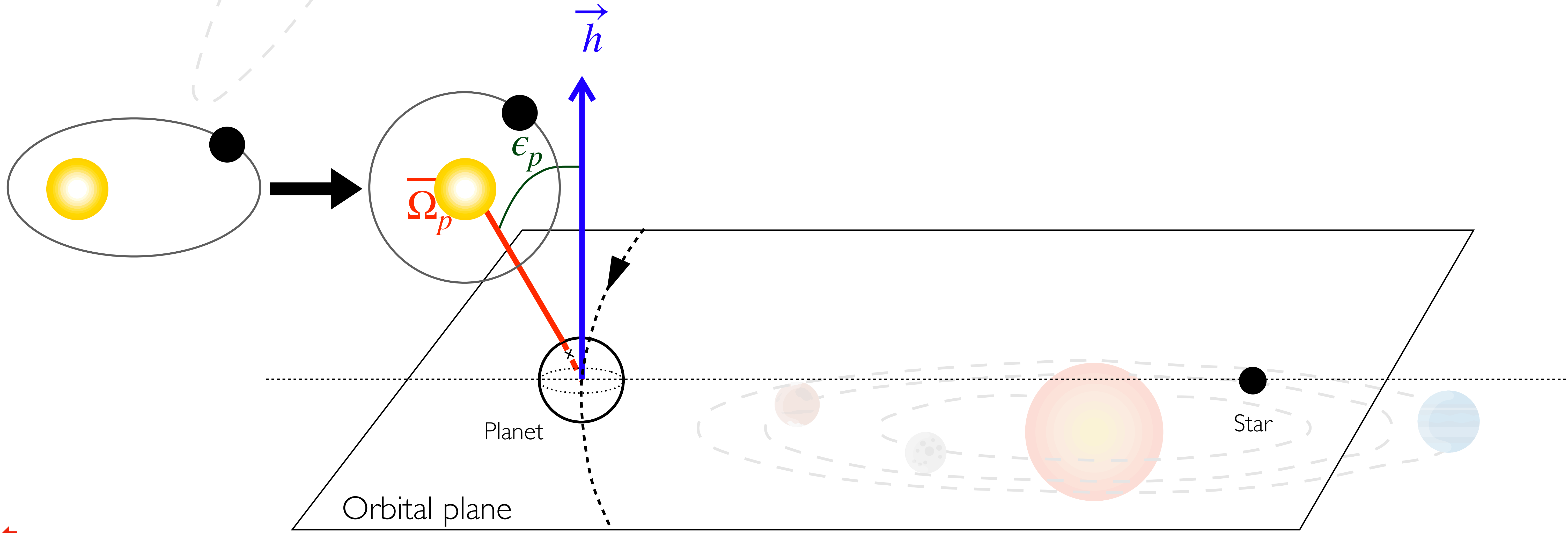


Ex: Mercury has $P_{rot} = 2/3\ P_{orb}$

Planetary tide



- **Circular** orbit: quick synchronization
- **Eccentric** orbit: quick pseudo-synchronization / spin-orbit resonance
- **Obliquity** of planet decreases
- **Eccentricity** of planet decreases
- Planet migrates inward
- Due to **deformation,** planet generates heat
- **Timescales** depend on **planetary radius** and the **planetary dissipation**



Planetary tide

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Low Q (high Δt) means a **fast evolution**

High Q (low Δt) means a **slow evolution**

Stars: $Q_\star \approx 10^5 - 10^8$ *[Penev, 2018]*

Earth: $Q_\oplus \approx 12 \ (\Delta t = 638 \ \mathrm{s})$ *[Goldreich & Soter 1966; Neron de Surgy & Laskar 97]*

Jupiter: $Q_{jup} \approx 3 \times 10^4$ *[e.g., Lainey+2009, for Io's frequency]*

Before we go more into details...

Any questions ?

Tidal interactions

- Why are tides important?
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  - ★ Tides, the hard way… 🫣
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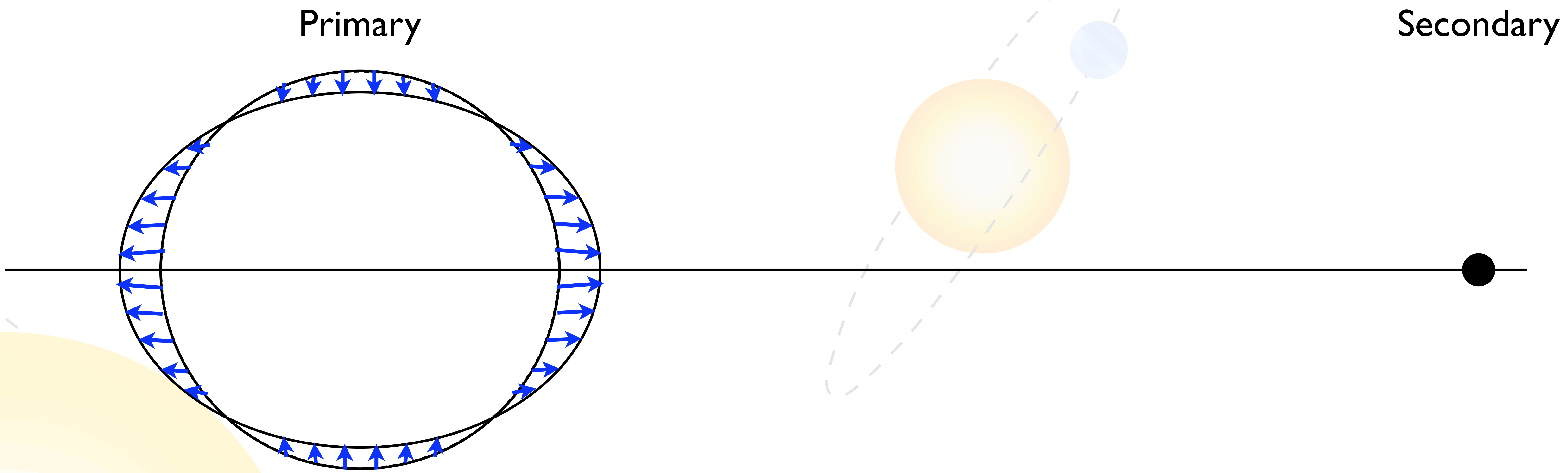
Tidal interactions



Tidal force **perturbs** the **hydrostatic balance**.
And this results in:

- A **mass redistribution**
- **Perturbations** of the **gravitational potential**

Tidal interactions

There are **two components** to tides:

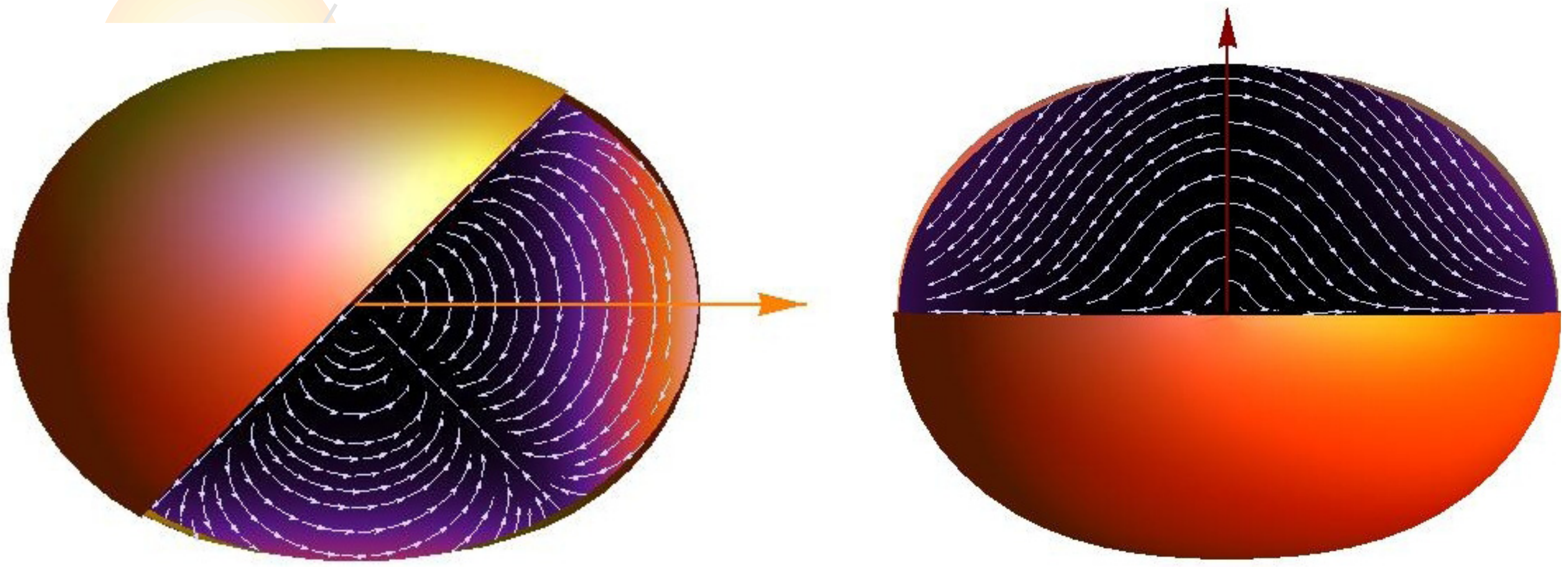
- The **equilibrium tide**

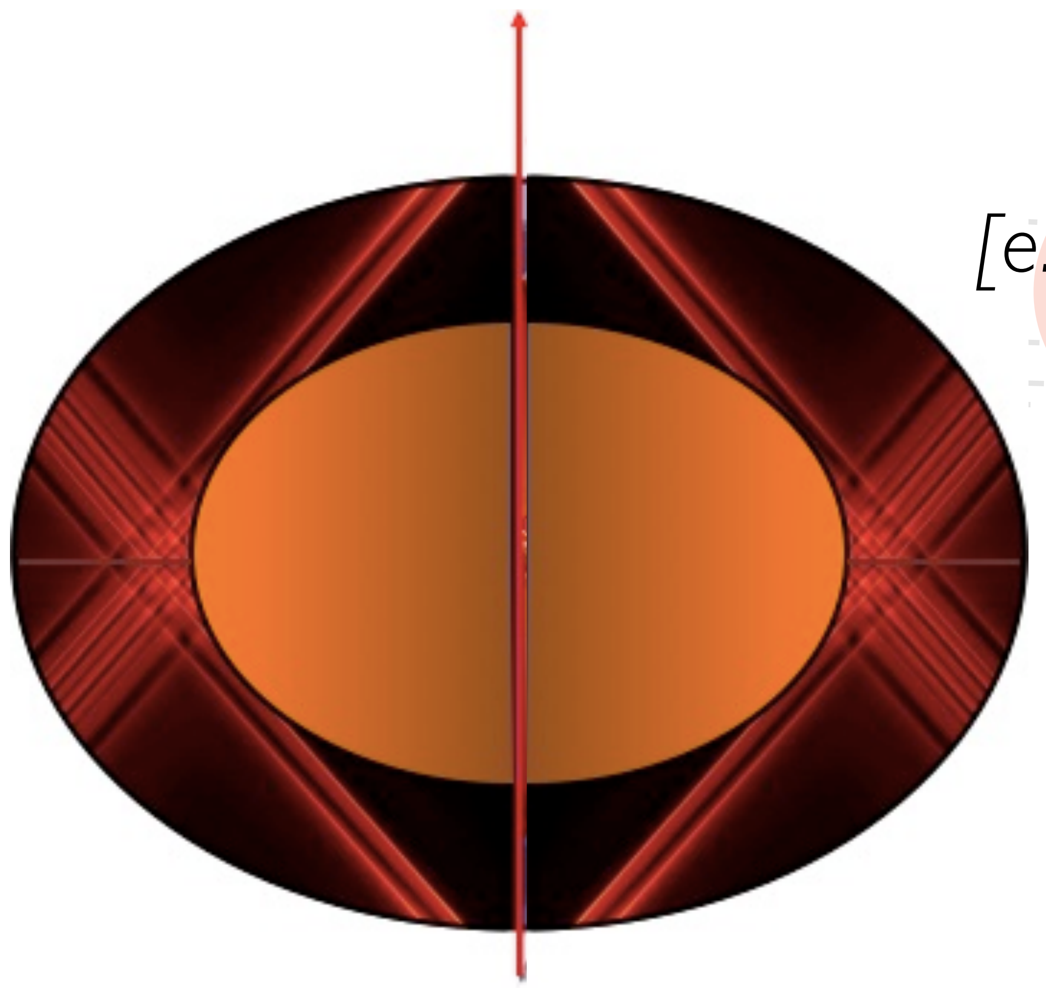
  Large-scale circulation resulting from the **hydrostatic adjustment** to the **tidal perturbation**

- The **dynamical tide**

  Fluid (elastic) eigenmodes of oscillations of the distorted body

[e.g. in a star: Remus+12a]

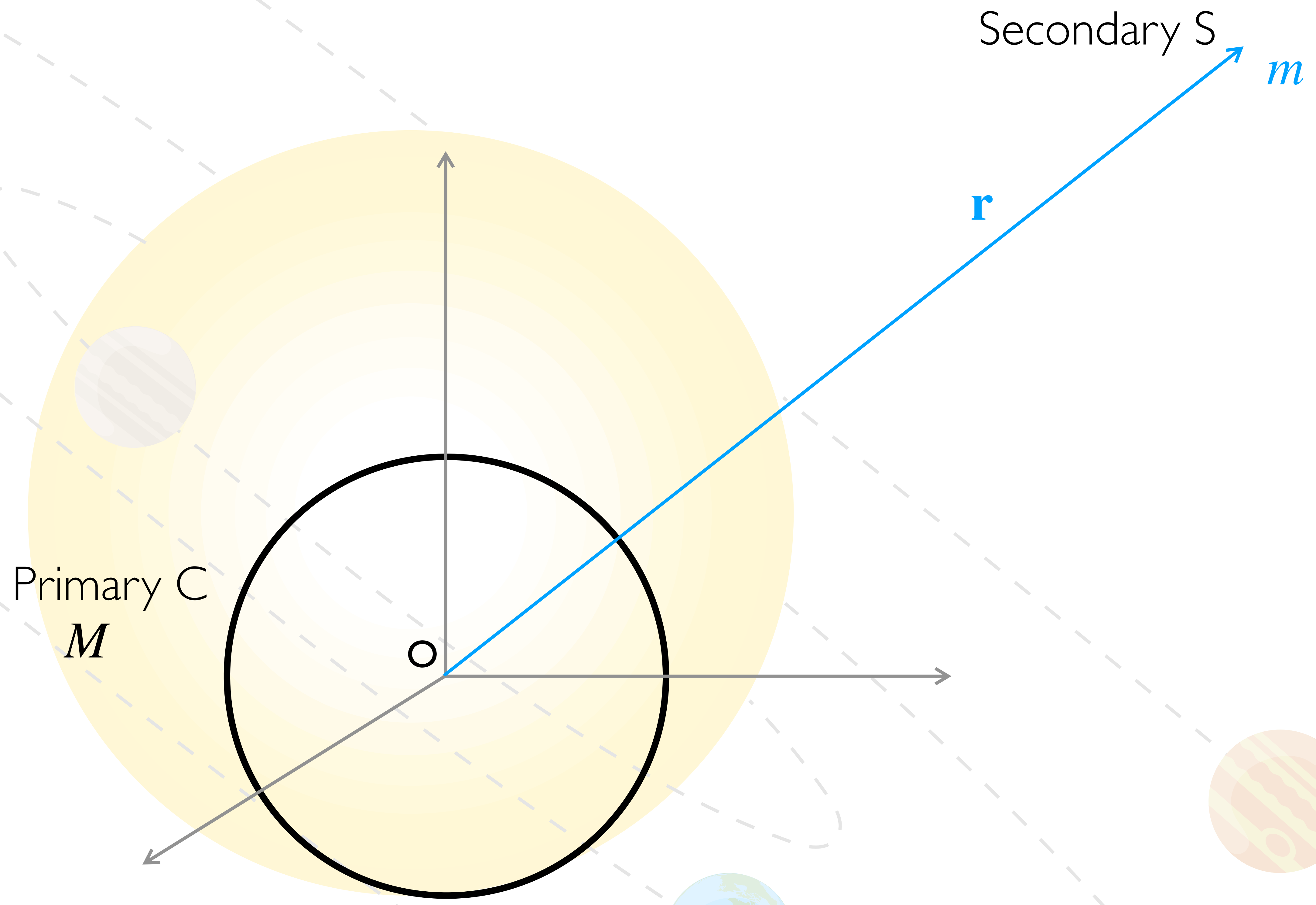




[e.g. in a gas giant: Guenel+16]

Tidal theory: Equilibrium tide

Let us consider a spherical body C (the primary) of mass M on presence of a second body S (secondary) of mass m



How does the primary **adjust its shape** so that **all forces** are **balanced**?
i.e. so that it is in **hydrostatic equilibrium**?

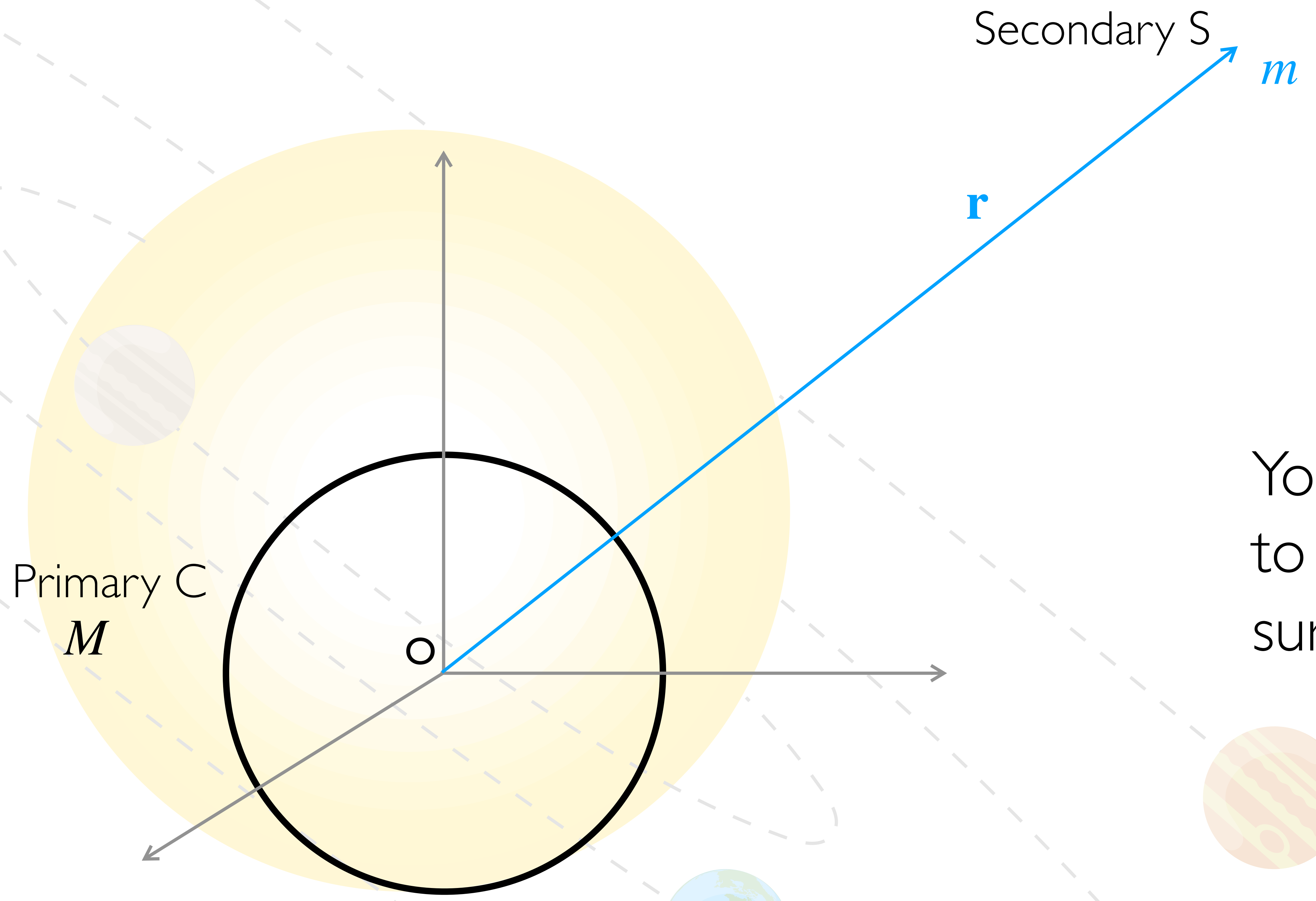
Hydrostatic equilibrium

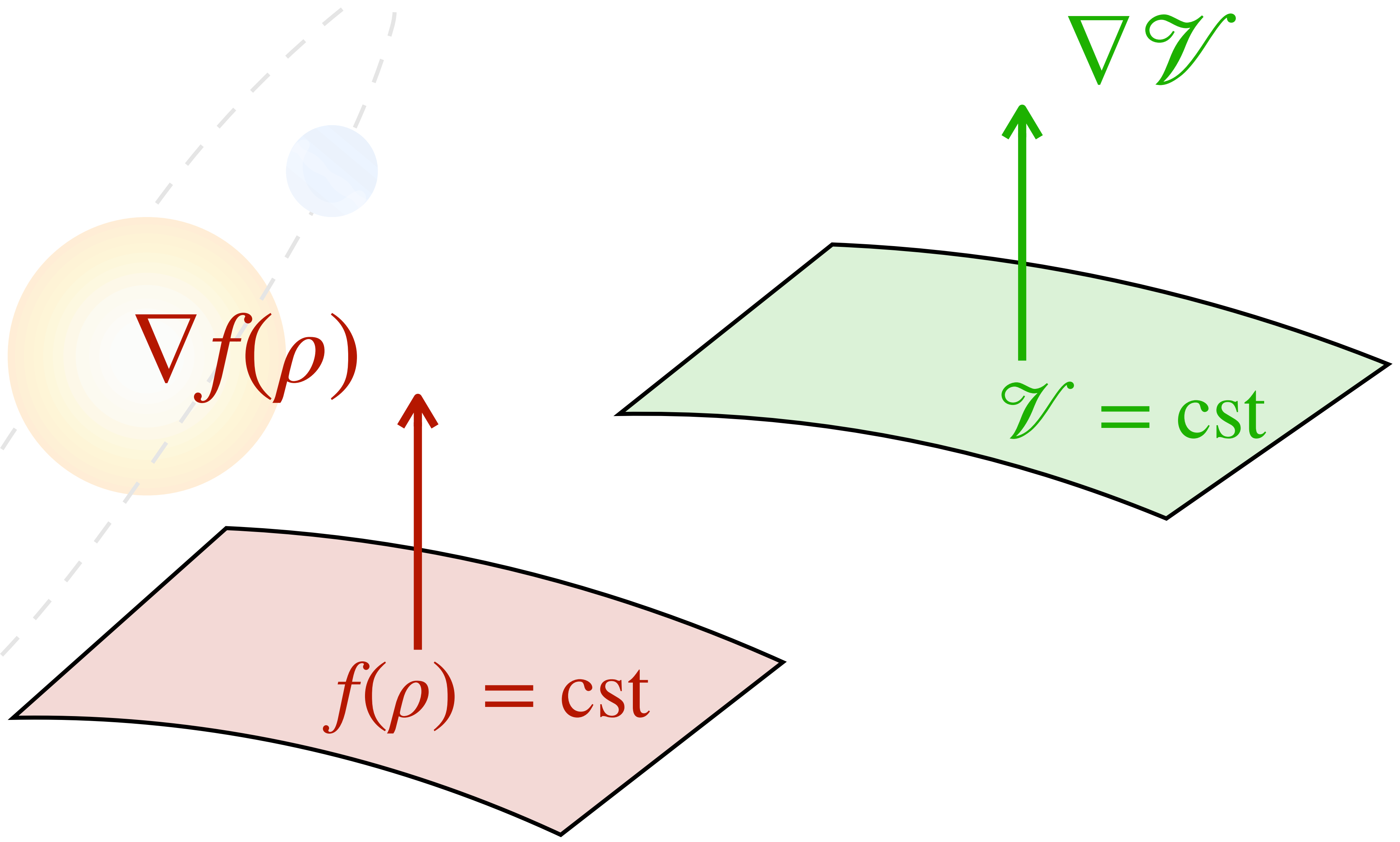
$$0 = -\frac{1}{\rho}\nabla P - \nabla \mathscr{V}$$

Gravitational potential

Pressure gradient force

Tidal theory: Equilibrium tide





You can show that the shape of the deformed primary is close to a shape for which **surfaces of constant ρ** coincides with surfaces of **surfaces of constant $\mathscr{V}$**

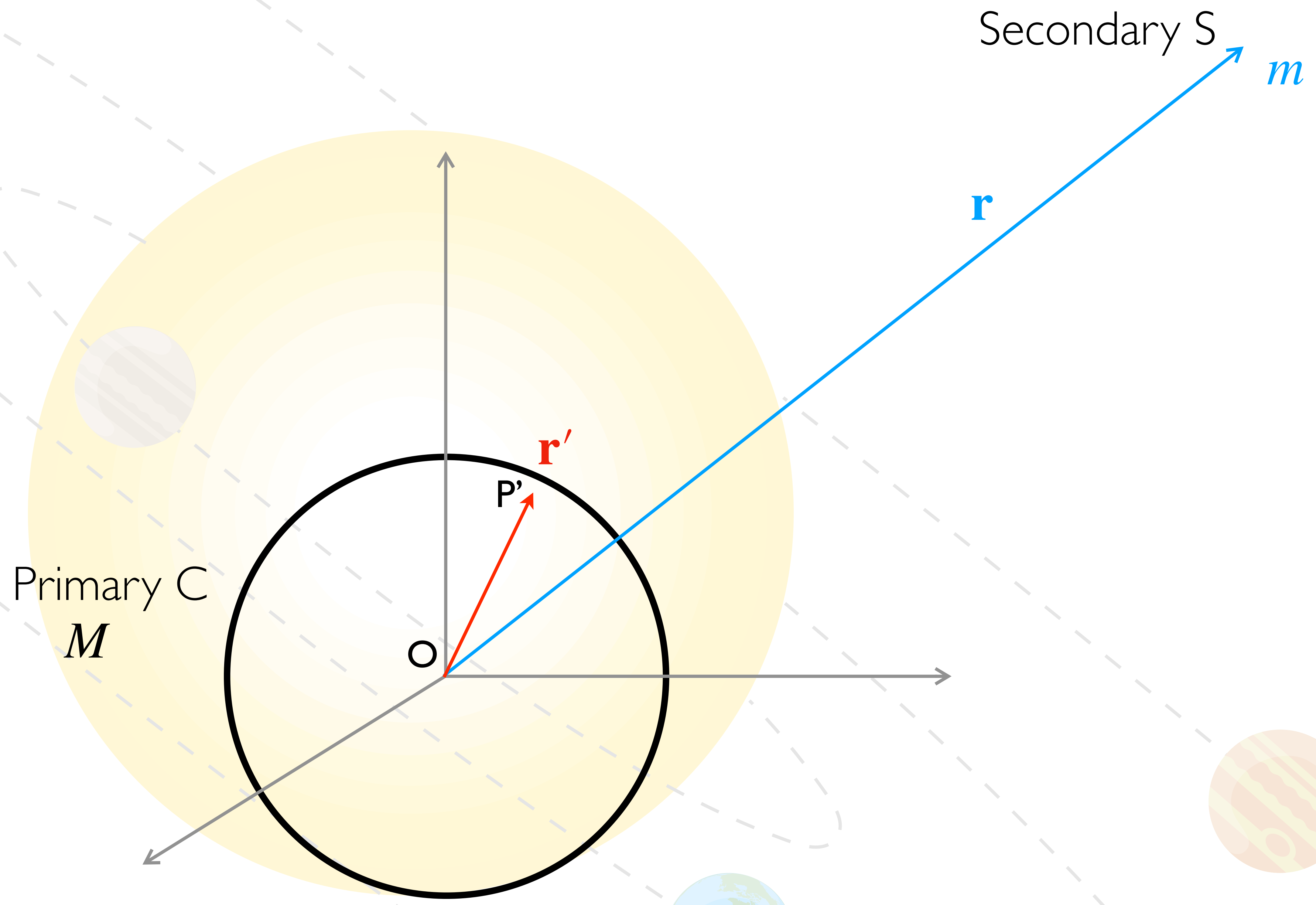
- Need to calculate $\mathscr{V}$

Tidal theory: perturbing potential

Let's calculate the **perturbing tidal potential**

Tidal theory: perturbing potential

Let us consider $\mathbf{r}$ the position of the secondary S of mass m, and $\mathbf{r}'$ the position of a point P' at the surface of the primary



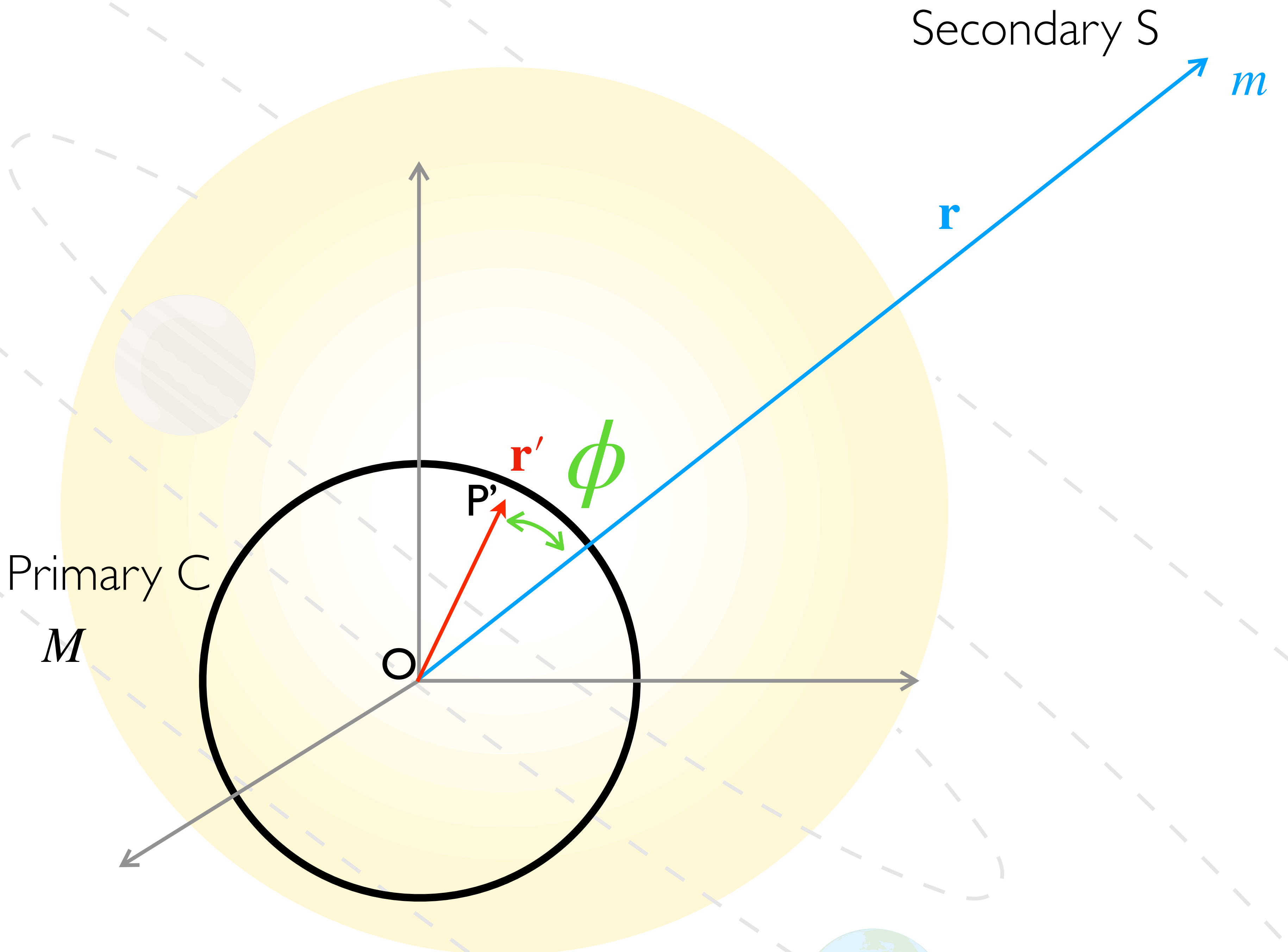
The **gravitational potential** $\mathscr{V}_S$ created by S at the point P' is given by:

$$\mathscr{V}_S(\mathbf{r}, \mathbf{r}') = -\frac{\mathscr{G}m}{\| \mathbf{r} - \mathbf{r}' \|}$$

$$= -\frac{\mathscr{G}m}{\sqrt{r^2 + r'^2 - 2(\mathbf{r} \cdot \mathbf{r}')}}$$

Tidal theory: perturbing potential

The **gravitational potential** $\mathscr{V}$ created by P at the point P' is given by: $\mathscr{V}(\mathbf{r}, \mathbf{r}') = -\dfrac{\mathscr{G}m}{\sqrt{r^2 + r'^2 - 2(\mathbf{r}\,.\,\mathbf{r}')}}$



The term $1/\sqrt{r^2 + r'^2 - 2(\mathbf{r}\,.\,\mathbf{r}')}$ can be rewritten using Legendre polynomials:

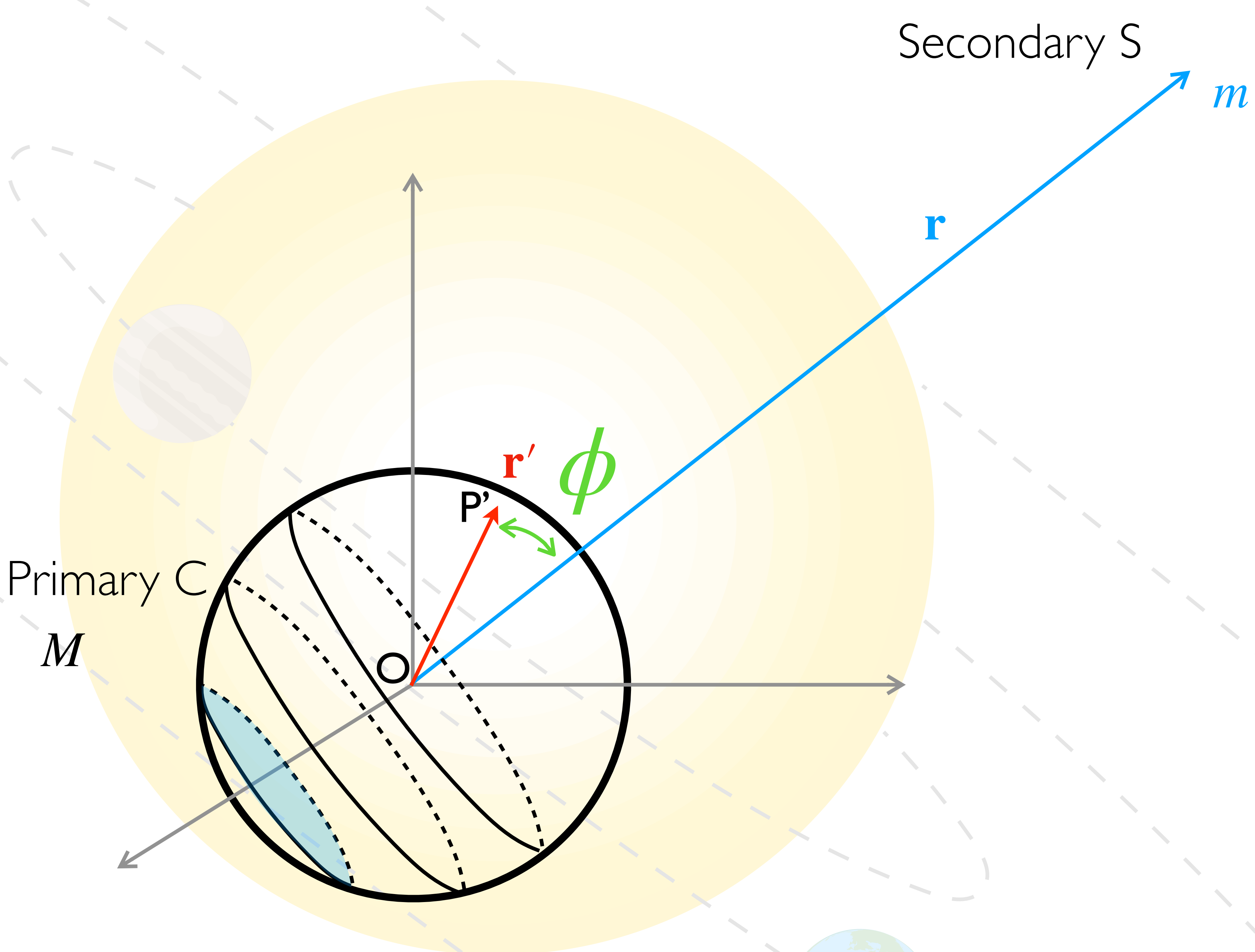
$$\mathscr{V}(\mathbf{r}, \mathbf{r}') = -\frac{\mathscr{G}m}{r}\sum_{l=0}^{\infty}\left(\frac{r'}{r}\right)^l P_l(\cos\phi)$$

$$= \sum_{l=0}^{\infty} V_l(r, r')$$

where ϕ is the angle between $\mathbf{r}$ and $\mathbf{r}'$.

Tidal theory: perturbing potential

The **gravitational potential** $\mathscr{V}$ created by P at the point P' is given by: $\mathscr{V}(\mathbf{r},\mathbf{r}') = \sum_{l=0}^{\infty} V_l(r,r')$



The first terms are given by:

$$\begin{cases} V_0(r,r') = -\dfrac{\mathscr{G}m}{r} \\ V_1(r,r') = -\dfrac{\mathscr{G}m}{r}\cos\phi\dfrac{r'}{r} \\ V_2(r,r') = -\dfrac{\mathscr{G}m}{r}P_2(\cos\phi)\left(\dfrac{r'}{r}\right)^2 = -\dfrac{\mathscr{G}m}{r}\left(\dfrac{3}{2}\cos^2\phi - 1\right)\left(\dfrac{r'}{r}\right)^2 \end{cases}$$

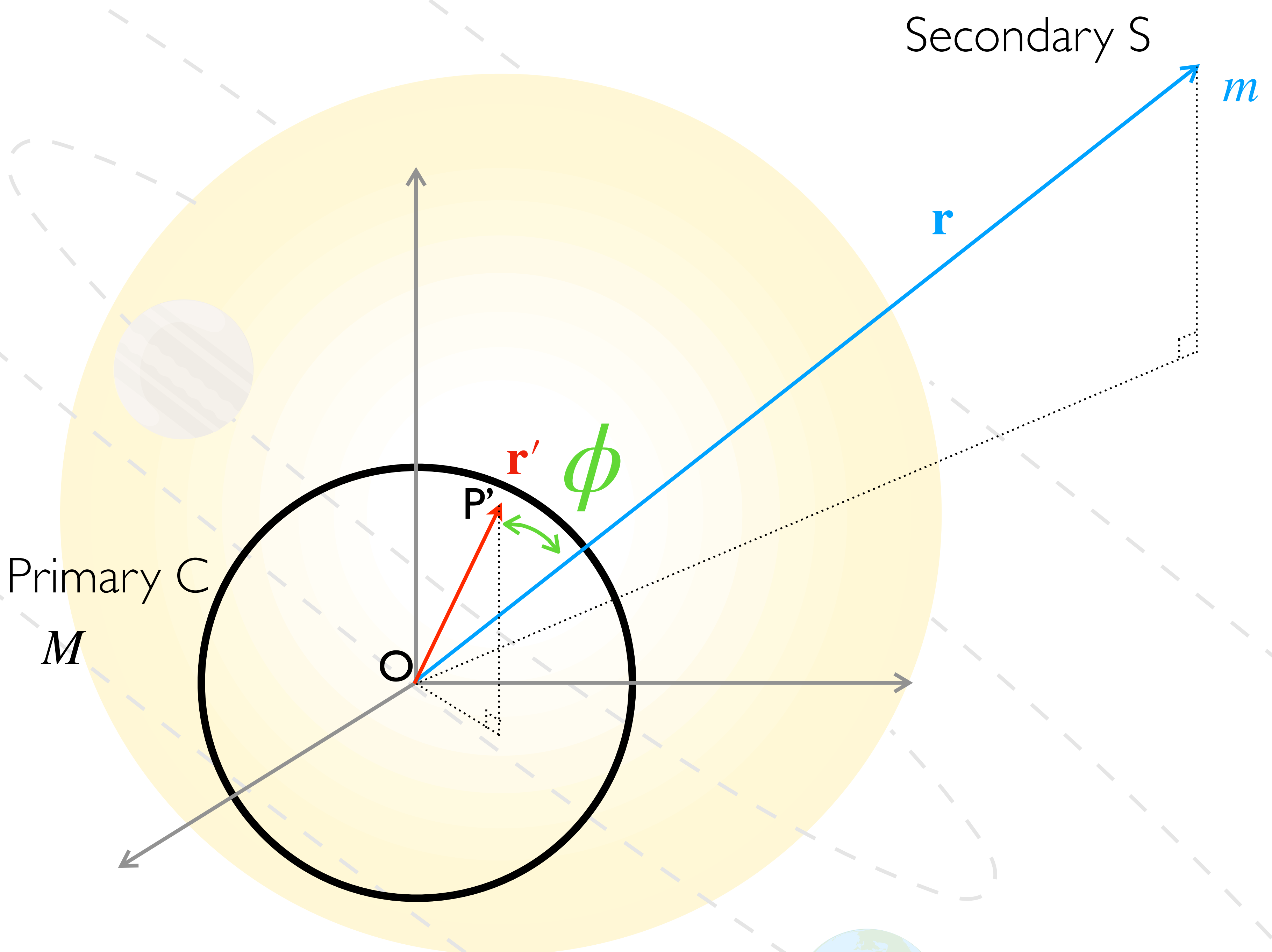
V_0 is constant in space so that $\mathbf{f} = -\nabla V_0$ is 0.

Responsible for Keplerian motion

Tidal theory: perturbing potential

The **tidal potential** is therefore given by:

$$\mathscr{V}_{\text{tid}}(\mathbf{r}, \mathbf{r}') = -\frac{\mathscr{G}m}{r}\sum_{l=2}^{\infty} P_l(\cos\phi)\left(\frac{r'}{r}\right)^l$$



In practice, we only consider the **quadrupolar** term $l = 2$

This is true if $r' \ll r$

The approximation is valid for $a > 5R_p$ and for small eccentricities

[Mathis & Le Poncin-Lafitte 2009]

$$\mathscr{V}_{\text{tid}}(\mathbf{r}, \mathbf{r}') = -\frac{\mathscr{G}m}{r}P_2(\cos\phi)\left(\frac{r'}{r}\right)^2 \propto \frac{1}{r^3}$$

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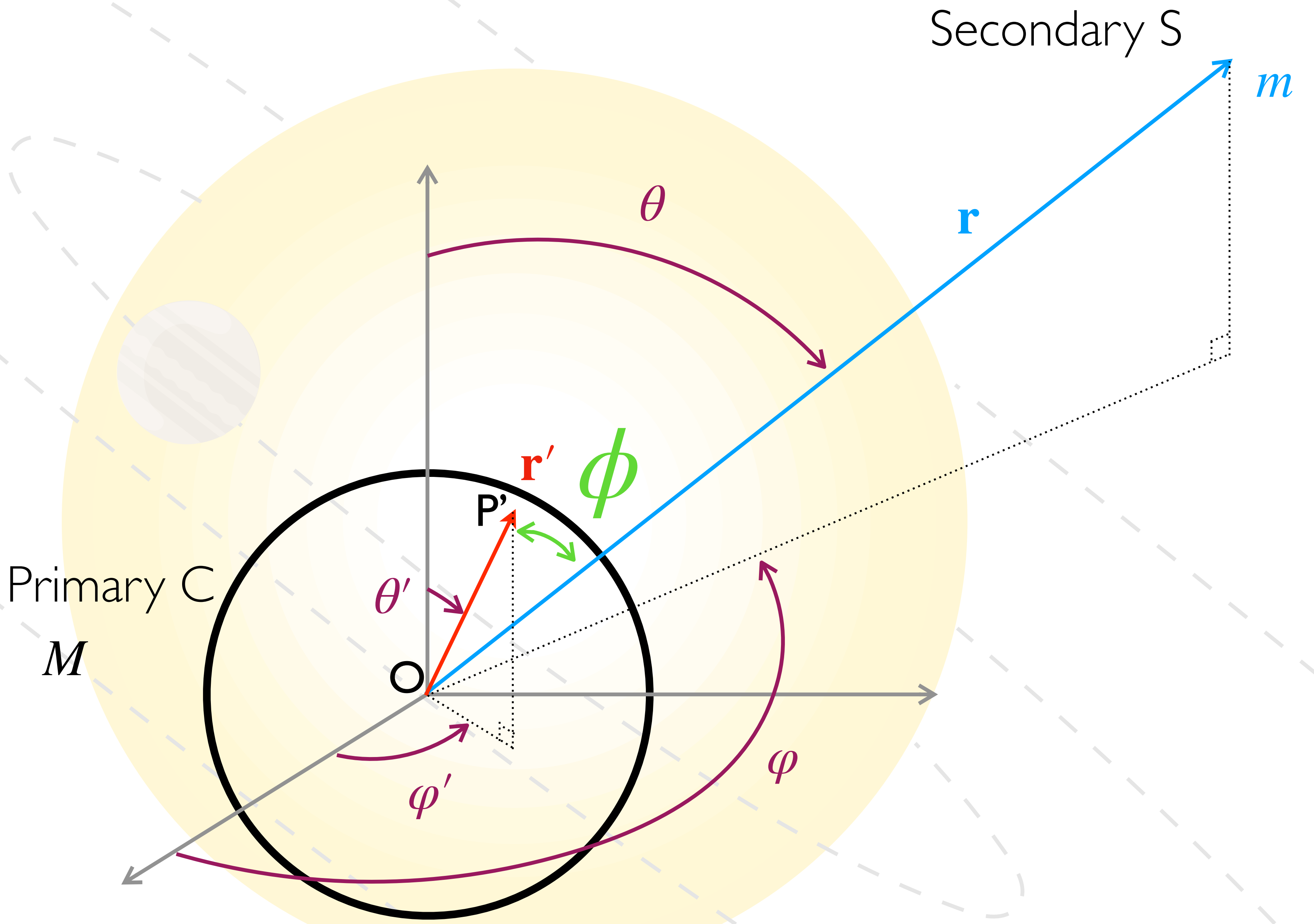

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$$\mathscr{V}_{\text{tid}}(\mathbf{r},\mathbf{r}') = -\frac{\mathscr{G}m}{r}P_2(\cos\phi)\left(\frac{r'}{r}\right)^2 \propto \frac{1}{r^3}$$

We can express $\cos\phi$ with the longitudes φ and φ' and colatitudes θ and θ' of P and P'

$$\cos\phi = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi - \varphi')$$ (addition theorem)

Tidal theory: perturbing potential

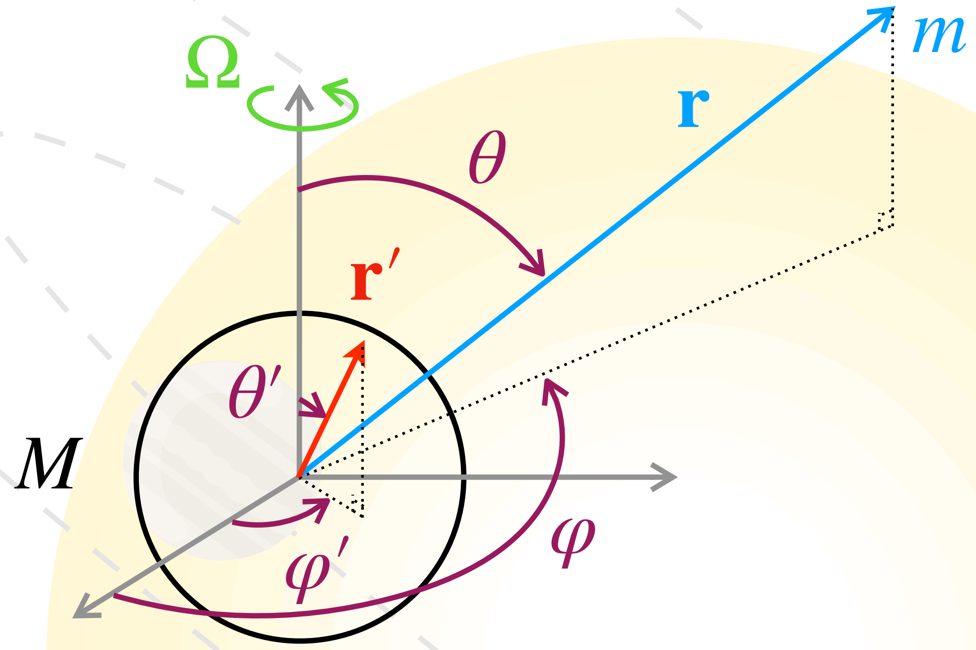
The **tidal potential** is therefore given by:

constant

$$\mathscr{V}_{\text{tid}}(\mathbf{r},\mathbf{r}') = -\frac{\mathscr{G}m}{r}\left(\frac{r'}{r}\right)^2 \times \left[\frac{1}{2}\left(3\cos^2\theta' - 1\right)\frac{1}{2}\left(3\cos^2\theta - 1\right) + \frac{3}{4}\sin^2\theta'\sin^2\theta\cos\left(2(\varphi-\varphi')\right) + \frac{3}{4}\sin 2\theta'\sin 2\theta\cos(\varphi-\varphi')\right]$$

Change with a frequency equal to the mean motion n

Change with a frequency equal to the primary spin Ω



For the Earth-Moon case ($\Omega \gg n$), we can see different components associated to different frequencies:

- The term in $\cos^2\theta = 1/2(1+\cos 2\theta)$ varies with frequency $2n$: it's the **fortnightly tide**
- The term in $\cos(\varphi - \varphi')$ varies with a frequency of $\Omega - n \approx \Omega$: it's the **diurnal tide**
- The term in $\cos(2(\varphi - \varphi'))$ varies with a frequency of $2(\Omega - n) \approx 2\Omega$: it's the **semi-diurnal tide**

Tidal theory: perturbing potential

- The term in $\cos(2(\varphi - \varphi'))$ varies with a frequency of $2(\Omega - n) \approx 2\Omega$: it's the semi-diurnal tide

2 high tides in one day

Tidal theory: perturbing potential

A convenient way of writing the potential comes from *Kaula [1962, 1964]*:
It allows to transform the **coordinates** of the secondary $(r,\ \theta,\ \varphi)$ in useful **dynamical parameters**:

- semi-major axis a,
- eccentricity e
- inclination I,
- argument of periastron ω^* ,
- argument of ascending node Ω^*

$$\mathcal{V}_{\text{tid}}(\mathbf{r},\mathbf{r}') = -\frac{\mathcal{G}m}{a}\sum_{l=2}^{\infty}\left(\frac{r'}{a}\right)^{l}\sum_{m=0}^{l}\frac{(l-m)!}{(l+m)!}(2-\delta_{0m})P_l^m(cos\theta')\sum_{p=0}^{l}\sum_{q\in\mathbb{Z}}F_{lmp}(I)G_{lpq}(e)$$

$$\left[\cos m\lambda'\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}_{l-m\ \text{odd}}^{2-m\ \text{even}}\left(\omega_{lmpq}t+(l-2p)\omega^*+m\Omega^*\right)\right.$$

$$\left.+\sin m\lambda'\begin{Bmatrix}\sin\\ -\cos\end{Bmatrix}_{2-m\ \text{odd}}^{2-m\ \text{even}}\left(\omega_{lmpq}t+(l-2p)\omega^*+m\Omega^*\right)\right] = \sum_{l=2}^{\infty}\sum_{m=0}^{l}\sum_{p=0}^{l}\sum_{q\in\mathbb{Z}}V_{lmpq}$$

ω_{lmpq} are the **frequencies of the forcing** and are given by: $\omega_{lmpq} = (l-2p+q)n - m\Omega$

Tidal theory: perturbing potential

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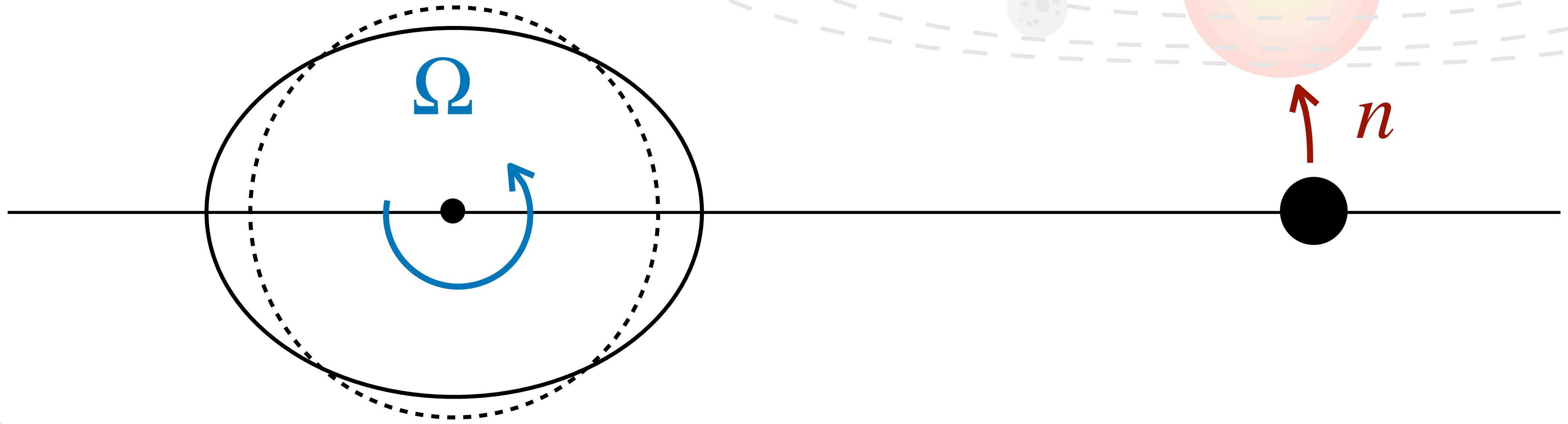
For $l = 2$, a **circular orbit**, and a **coplanar orbit**, there is one excitation frequency given by:

$$\omega_{lmpq} = 2(n - \Omega)$$

(it's the semi-diurnal frequency)



Tidal theory: perturbing potential

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$$\omega_{lmpq} = 2(n - \Omega)$$

(it's the semi-diurnal frequency)

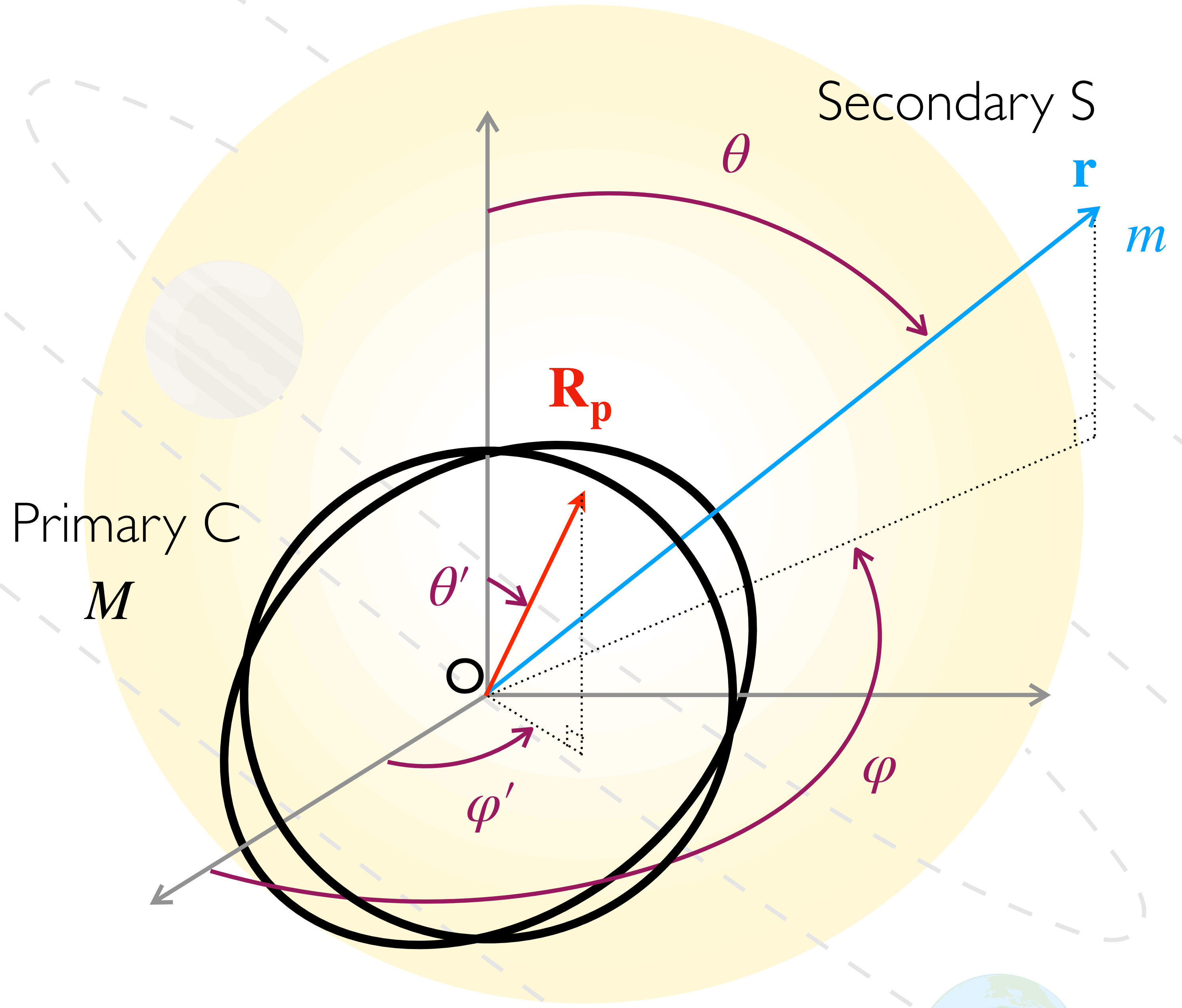
For an **eccentric orbit** or for an **inclined orbit**, additional frequencies are excited

Tidal theory: deformed body potential

Let's calculate the **potential** created by **deformed body**

Tidal theory: deformed body potential

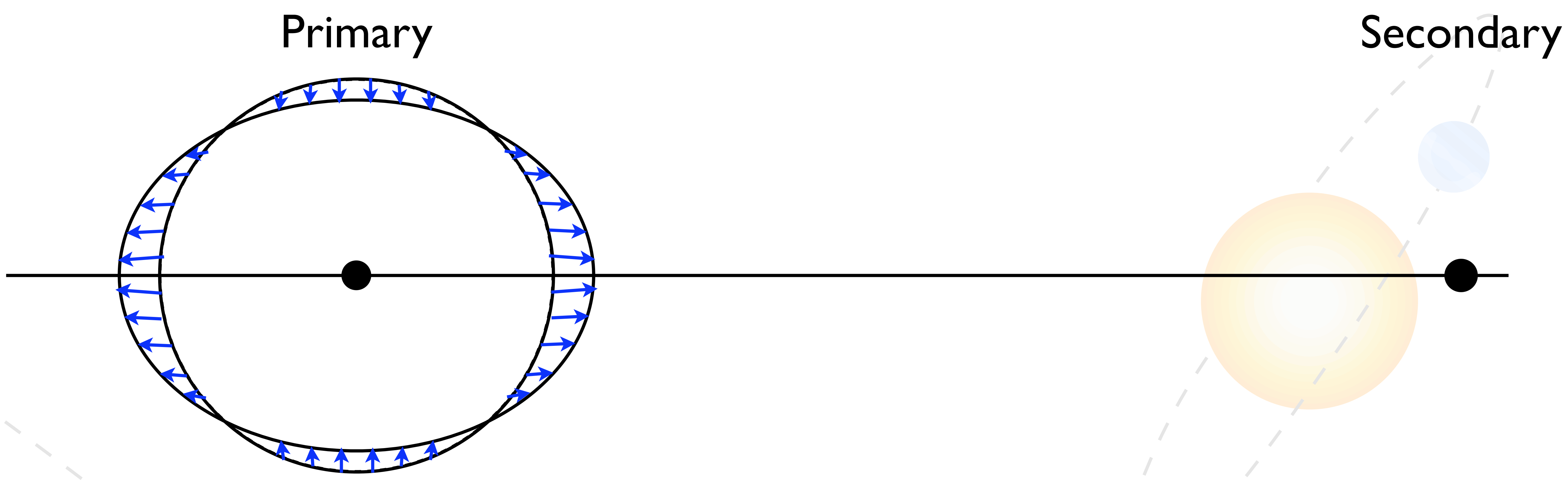
We use here the **theory of Love**:
The **potential** of the **deformed body** Φ at its surface is **proportional** to the corresponding component of the **perturbing potential** $\mathscr{V}_{\rm tid}$ at its surface *[Love, 1911]*.



$$\Phi_{\text{deformed body}}(r = R_{\text{surface}}) = k_2(\omega) \times \Phi_{\text{secondary}}(r = R_{\text{surface}})$$

Response function
(depends on properties of primary)

Tidal theory: deformed body potential



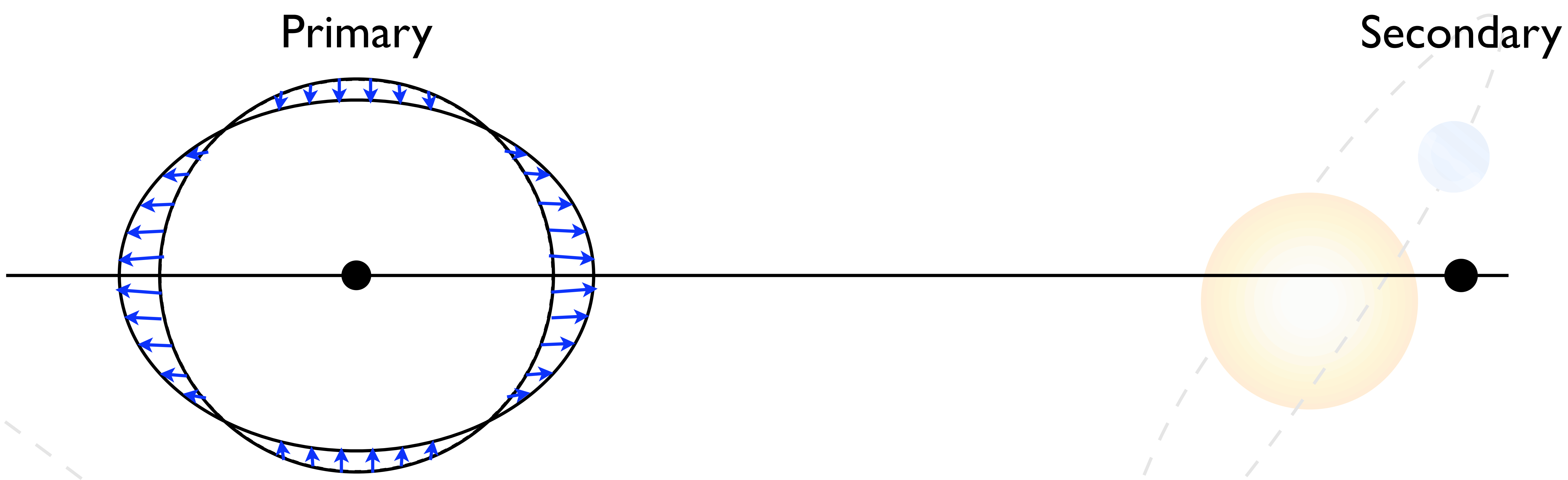
$$k_l(\omega_{lmpq}) = \mathrm{Re}k_l(\omega_{lmpq}) + i\ \mathrm{Im}k_l(\omega_{lmpq})$$

The tidal response can be divided into **two components:**

- Effects associated by the non-spherical shape of the distorted body: instantaneous response (non-dissipative)

  Responsible for orbital precession

Tidal theory: deformed body potential



$$k_l(\omega_{lmpq}) = \mathrm{Re}k_l(\omega_{lmpq}) + i\ \mathrm{Im}k_l(\omega_{lmpq})$$

The tidal response can be divided into **two components:**

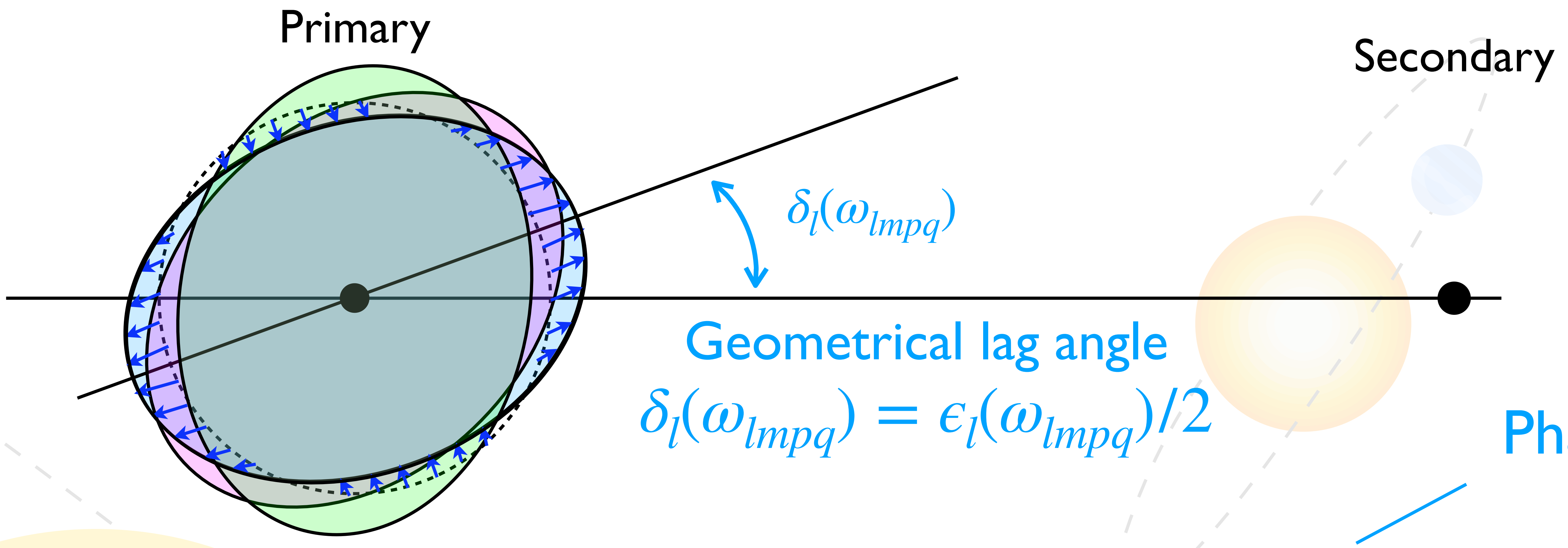
- Effects associated by the **non-spherical shape** of the distorted body: **instantaneous** response (non-dissipative)

  Responsible for **orbital precession**

- Effects associated by the **viscosity/rheology** of the distorted body: **delayed** response (dissipative)

  Responsible for **orbital** and **rotational evolution**

Tidal theory: deformed body potential



Geometrical lag angle

$$\delta_l(\omega_{lmpq}) = \epsilon_l(\omega_{lmpq})/2$$

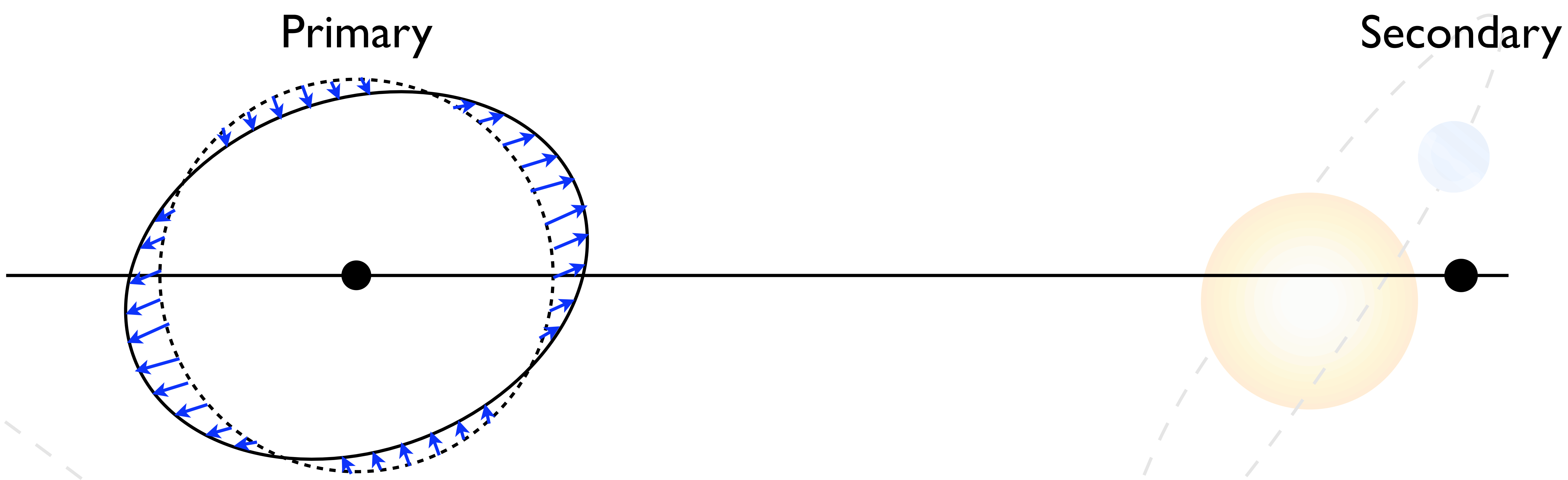
Phase lag

$$k_l(\omega_{lmpq}) = |k_l(\omega_{lmpq})| e^{-i\epsilon_l(\omega_{lmpq})}$$

$$-\mathrm{Im}k_l(\omega_{lmpq}) = |k_l(\omega_{lmpq})| \sin \epsilon_l(\omega_{lmpq})$$

Natural way to express **dissipation**

Tidal theory: deformed body potential



Constant phase lag

Using one **tidal quality factor** Q is equivalent of doing **many approximations**: in particular the **phase lag** $\epsilon_2(\omega) = \epsilon = \text{cst}$ *[Goldreich 1963]*

Constant time lag

Using one **time lag** Δt is equivalent of doing **many approximations**: in particular the **phase lag** $\epsilon_2(\omega) \propto \omega$ *[Darwin 1879]*

The **phase lag** has a **smooth dependency** in the **excitation frequency**

Equilibrium tide

➡ **Appropriate** for objects made of **weakly viscous fluid**

Tidal interactions

There are **two components** to tides:

- The **equilibrium tide**

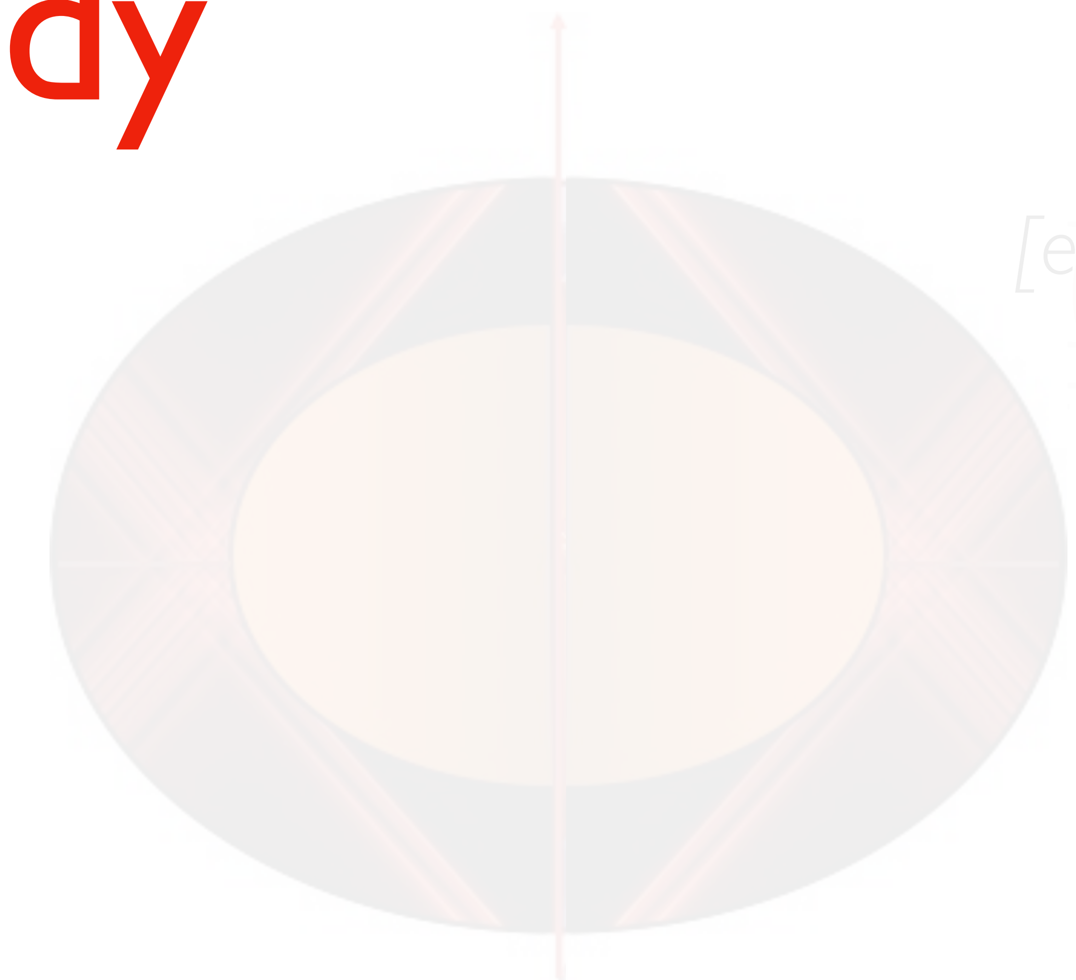
  Large-scale circulation resulting from the **hydrostatic adjustment** to the **tidal perturbation**

- The **dynamical tide**

  Fluid (elastic) eigenmodes of oscillations of the distorted body

[e.g. in a star: Remus+12a]

[e.g. in a gas giant: Guenel+16]



Tidal response depends on the properties of the extended body

Equilibrium vs dynamical tide

There are **two components** to tides:

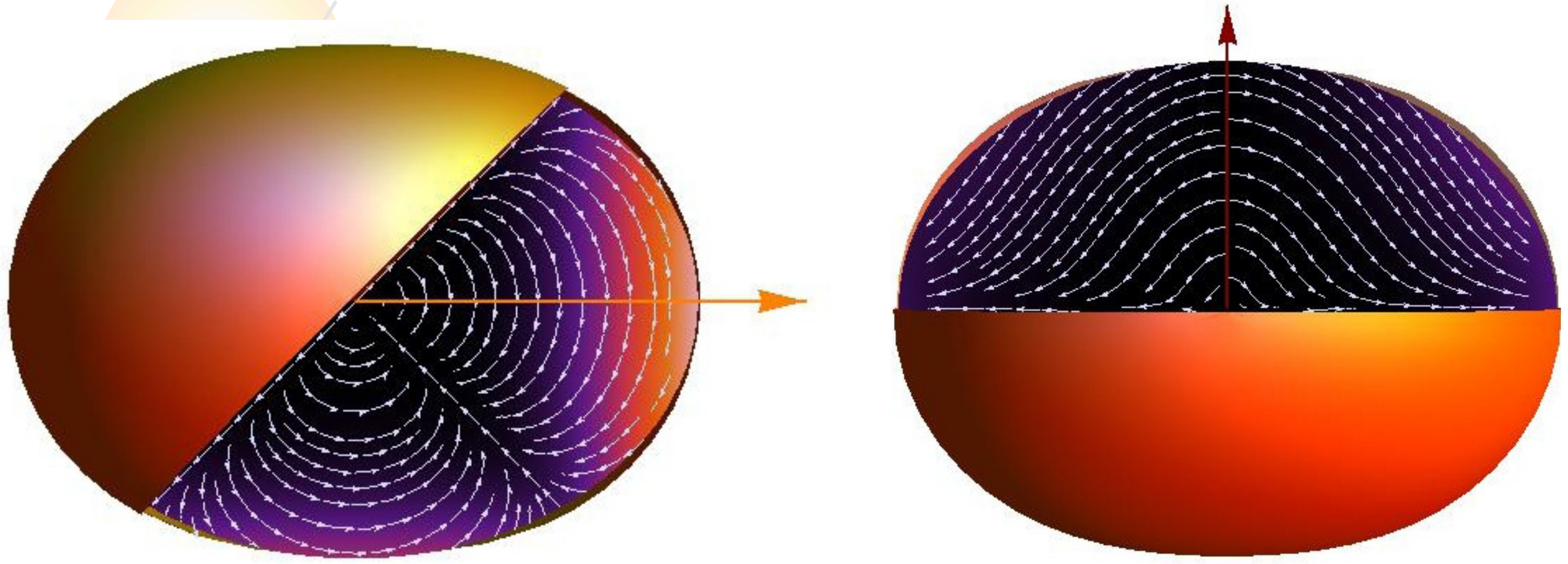
- The **equilibrium tide**

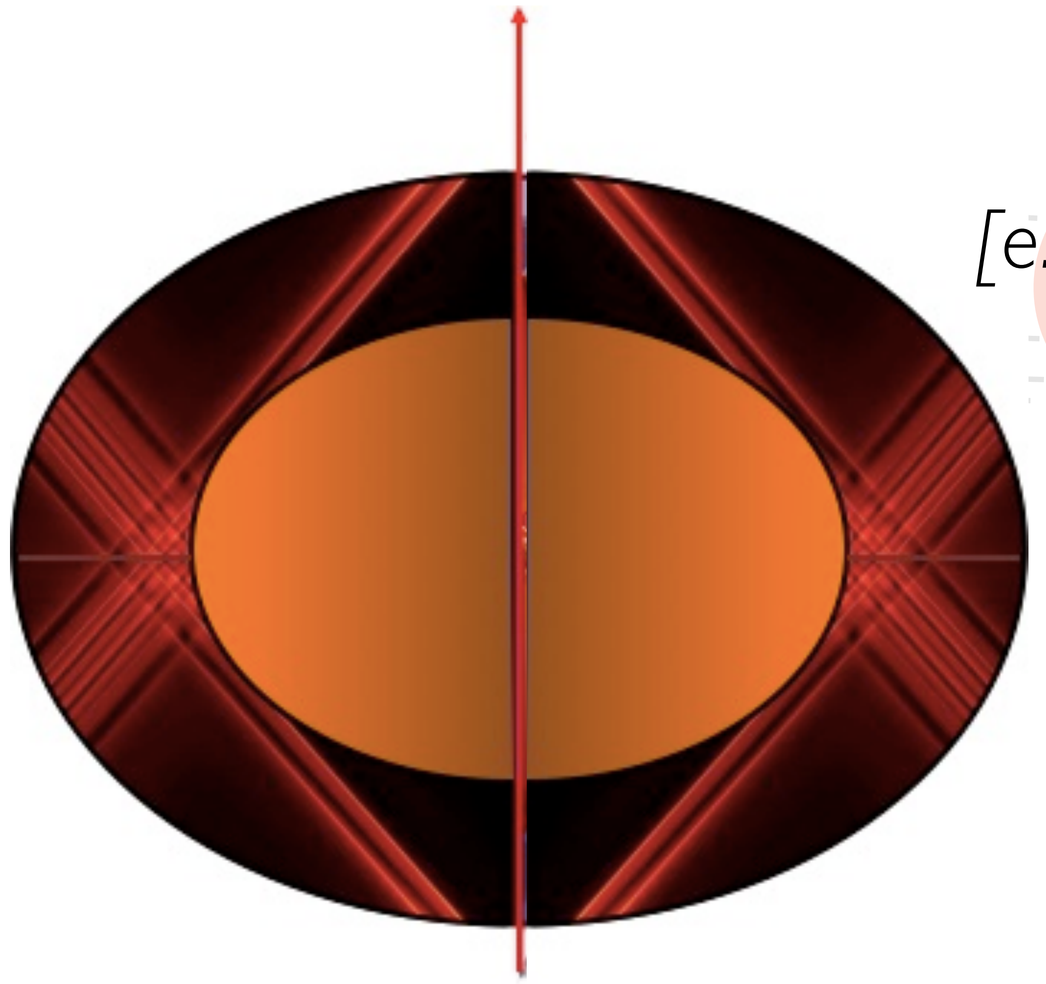
  Large-scale circulation resulting from the **hydrostatic adjustment** to the **tidal perturbation**

- The **dynamical tide**

  Fluid (elastic) eigenmodes of oscillations of the distorted body

[e.g. in a star: Remus+12a]

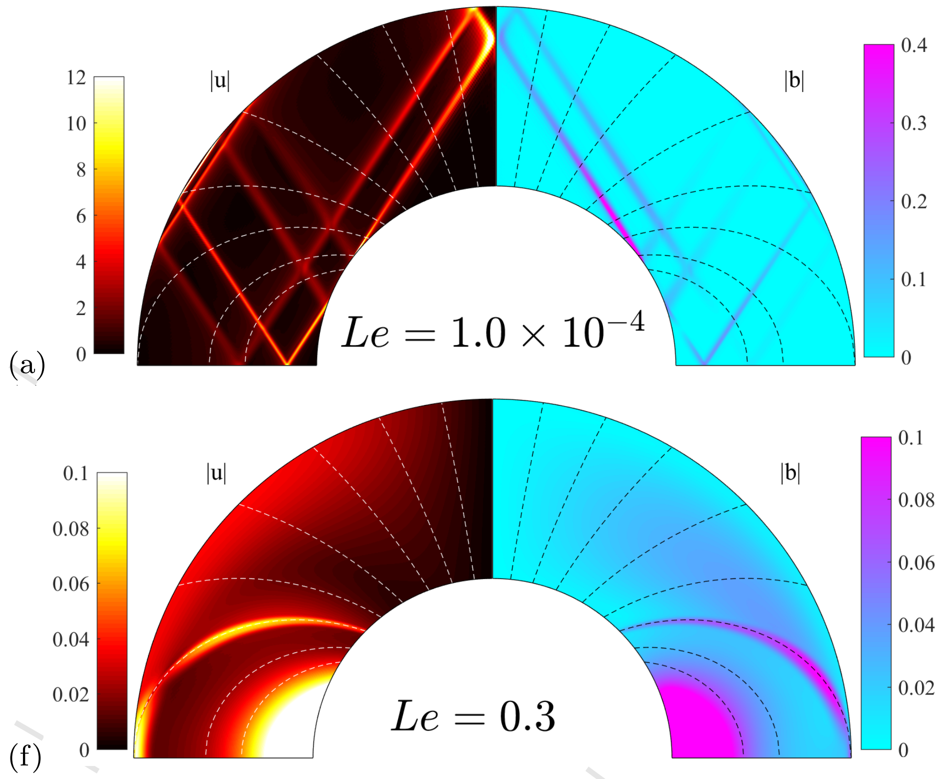




[e.g. in a gas giant: Guenel+16]

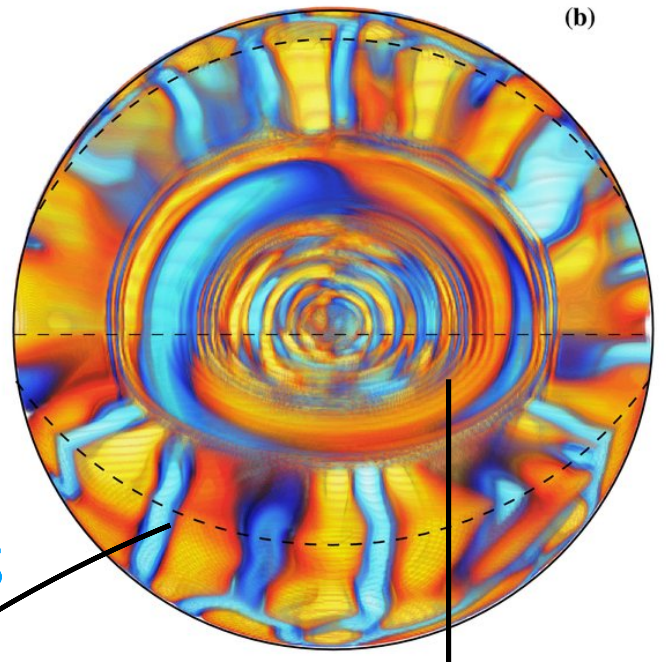
Dynamical tide

Adapted from
Mathis&Remus 2013



[Lin&Ogilvie18, Astoul+19]

[Brun+11]



Turbulent friction (viscous diss.)

i.e., convection zone of Sun-like stars/gas giants
(dynamics driven by Coriolis acc.)

Heat diffusion (thermal diss.)

i.e., radiative zone of Sun-like stars/oceans
(dynamics driven by buoyancy acc.)

High frequency waves: tides only a perturbation

Ohmic diss.

0 — ω_A — 2Ω — N — f_L — ω

Alfvén waves

Inertial waves

Gravity waves

Acoustic waves

2Ω is the **inertial frequency**

ω_A is the **Alfvén frequency**

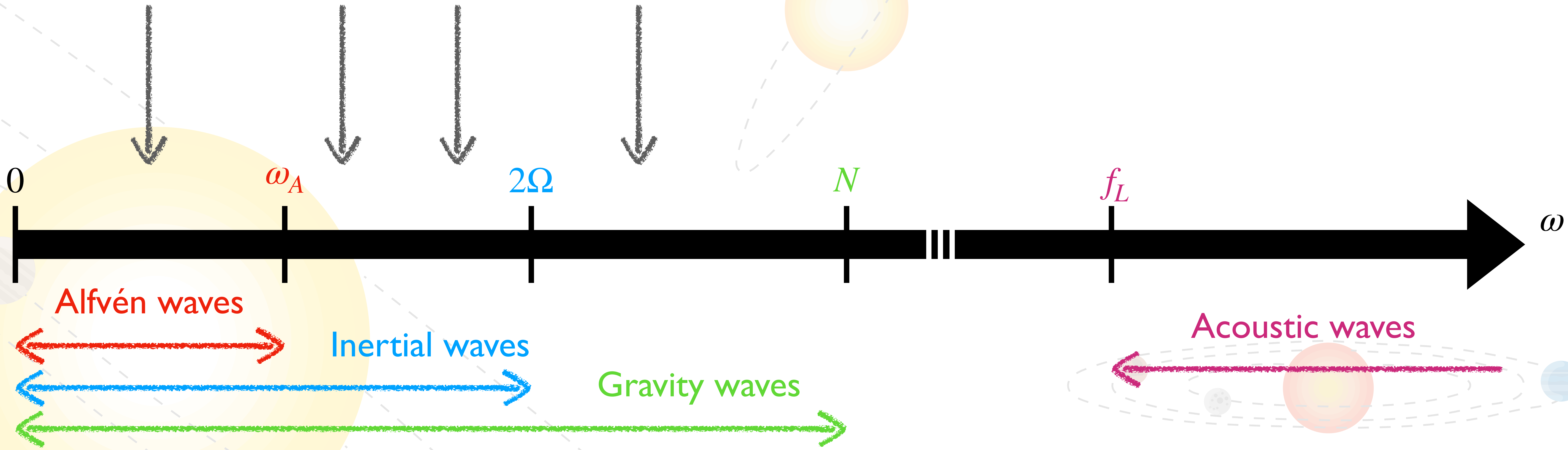
f_L is the **Lamb's frequency**

N is the **Brunt-Väisälä frequency** (or buoyancy frequency)

Dynamical tide

Adapted from
Mathis&Remus 2013

Excitation by each Fourier component of the potential



f_L is the **Lamb's frequency**

2Ω is the **inertial frequency**

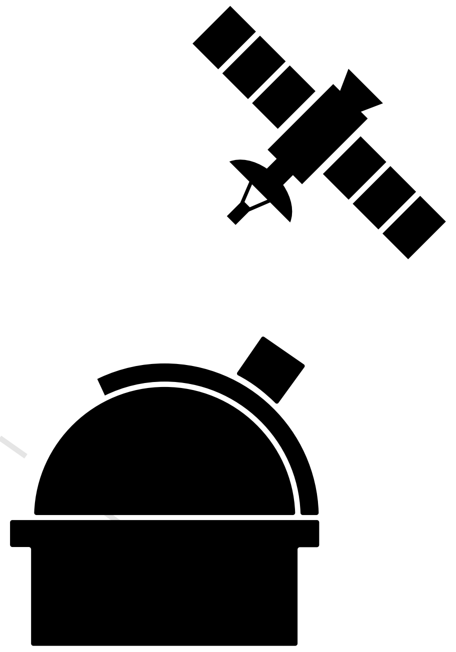
ω_A is the **Alfvén frequency**

N is the **Brunt-Väisälä frequency** (or buoyancy frequency)

Tidal interactions

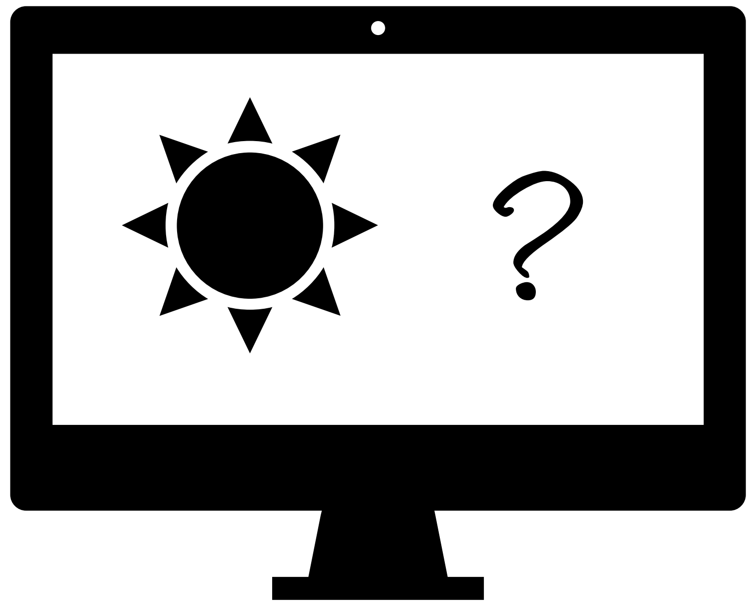
- Why are tides important?
- A bit of theory
  - ★ Tides, the easy way
  - ★ Tides, the hard way…
- Some constraints brought by studying tides…
  - ★ Stars
  - ★ Planets
  - ★ Multi-planet systems

Stellar tide



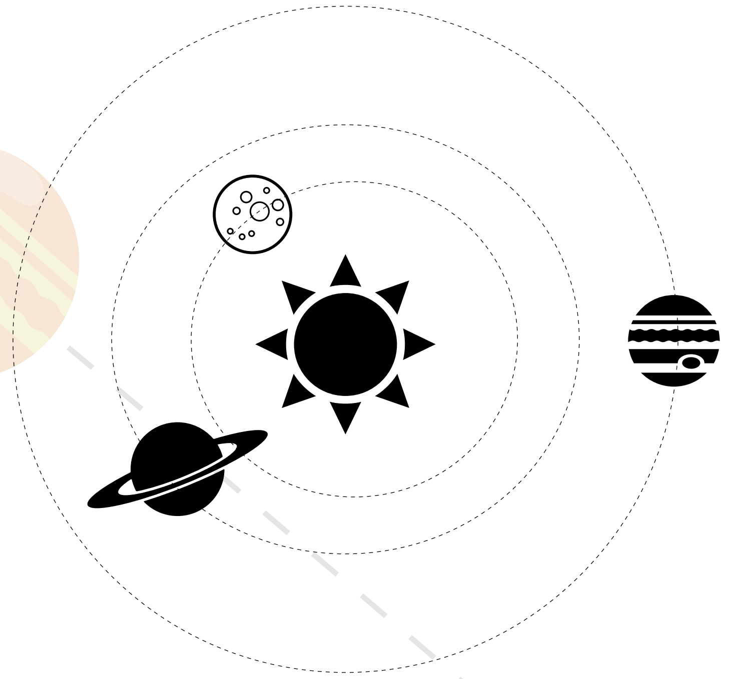
Stellar parameters

Internal structure



Tidal response

Orbital/rotational evolution



Stellar tide

Observational constraints

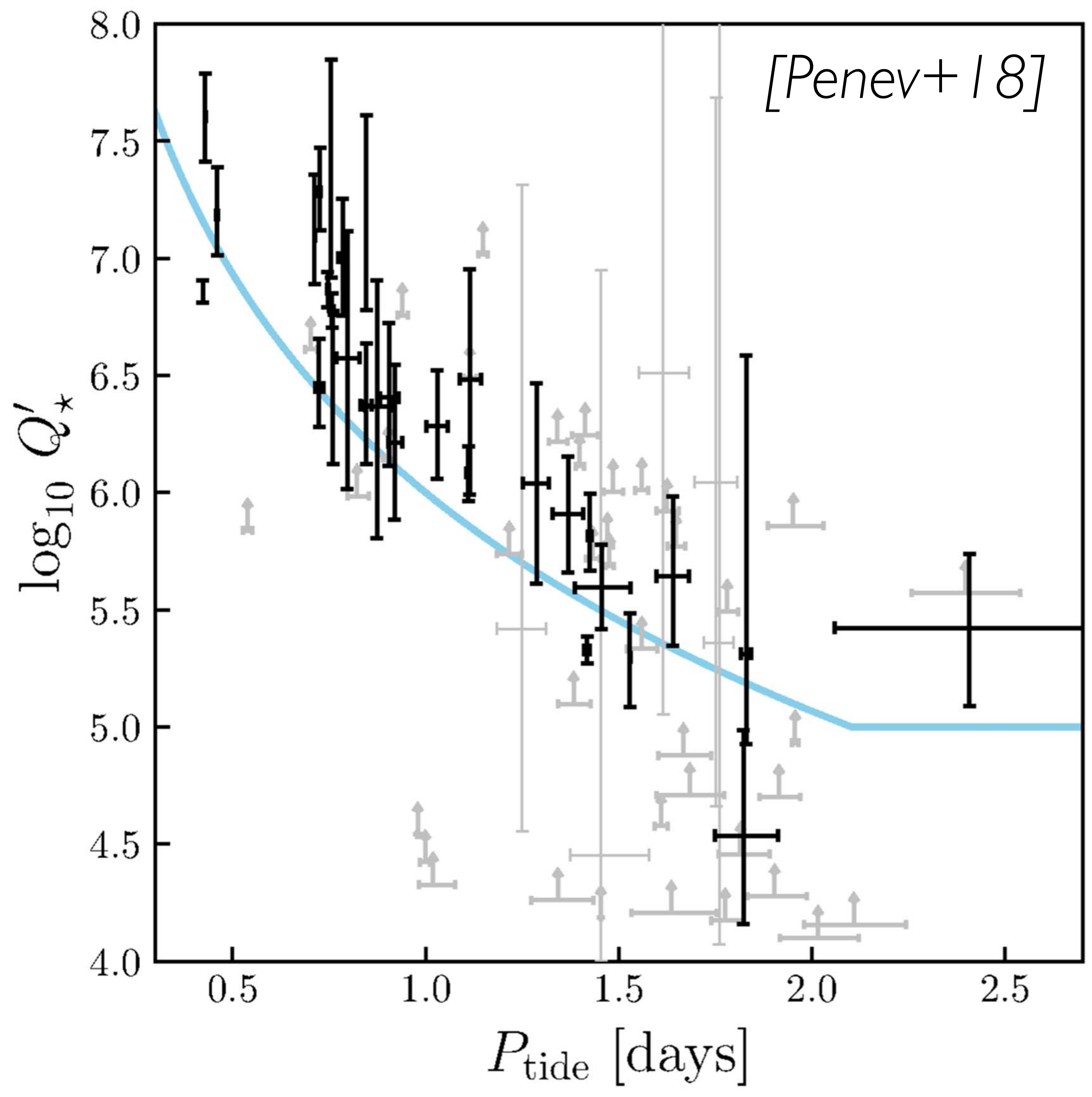
- *Meibom & Mathieu [2005]* used the tidal circularization of **binaries** in an open cluster to **estimate** $Q'_\star \approx 10^6$

- *Jackson+2008* used the tidal circularization of a small sample of exoplanets and found a best fitting value of $Q'_\star = 10^{5.5}$ (they also fit a planetary Q_p)

- *Collier Cameron & Jardine 2018* used the orbital distance distribution of HJs to calculate $\log_{10} Q'_\star = 8.26 \pm 0.14$ for the **equilibrium tide** regime, but a smaller value of $\log_{10} Q'_\star = 7.3 \pm 0.4$ for the **dynamical tide** regime

- Using the fact that inward **migration** of a massive planet leads to a spin up of the star, *Carone & Pätzold [2007]* analyzed the system OGLE-TR-56 and found $Q'_\star > 2 \times 10^7$

  Penev+18 also used this phenomenon for a statistical study of HJ hosts and find that $Q'_\star$ depends on the forcing frequency

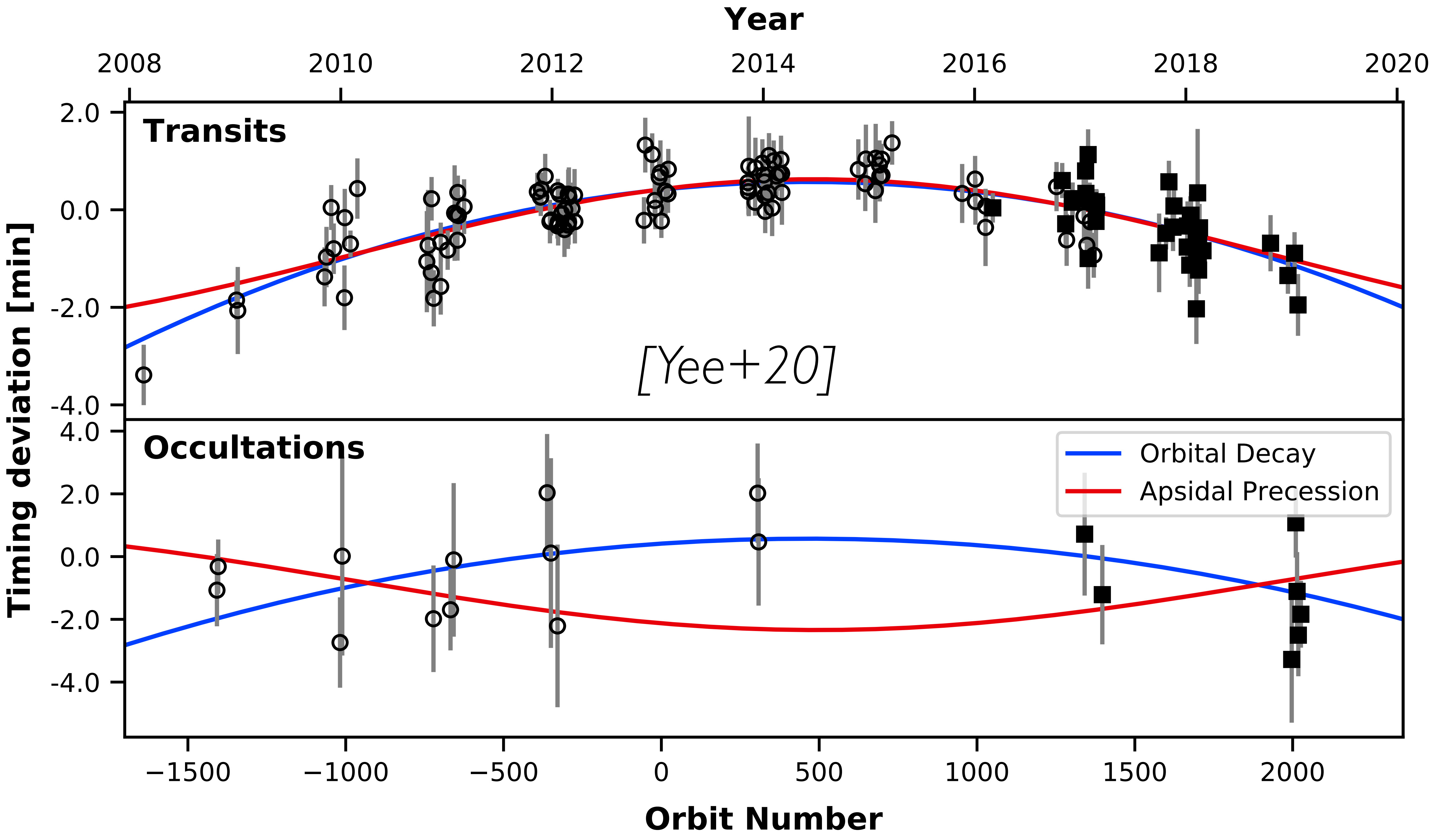


Stellar tide

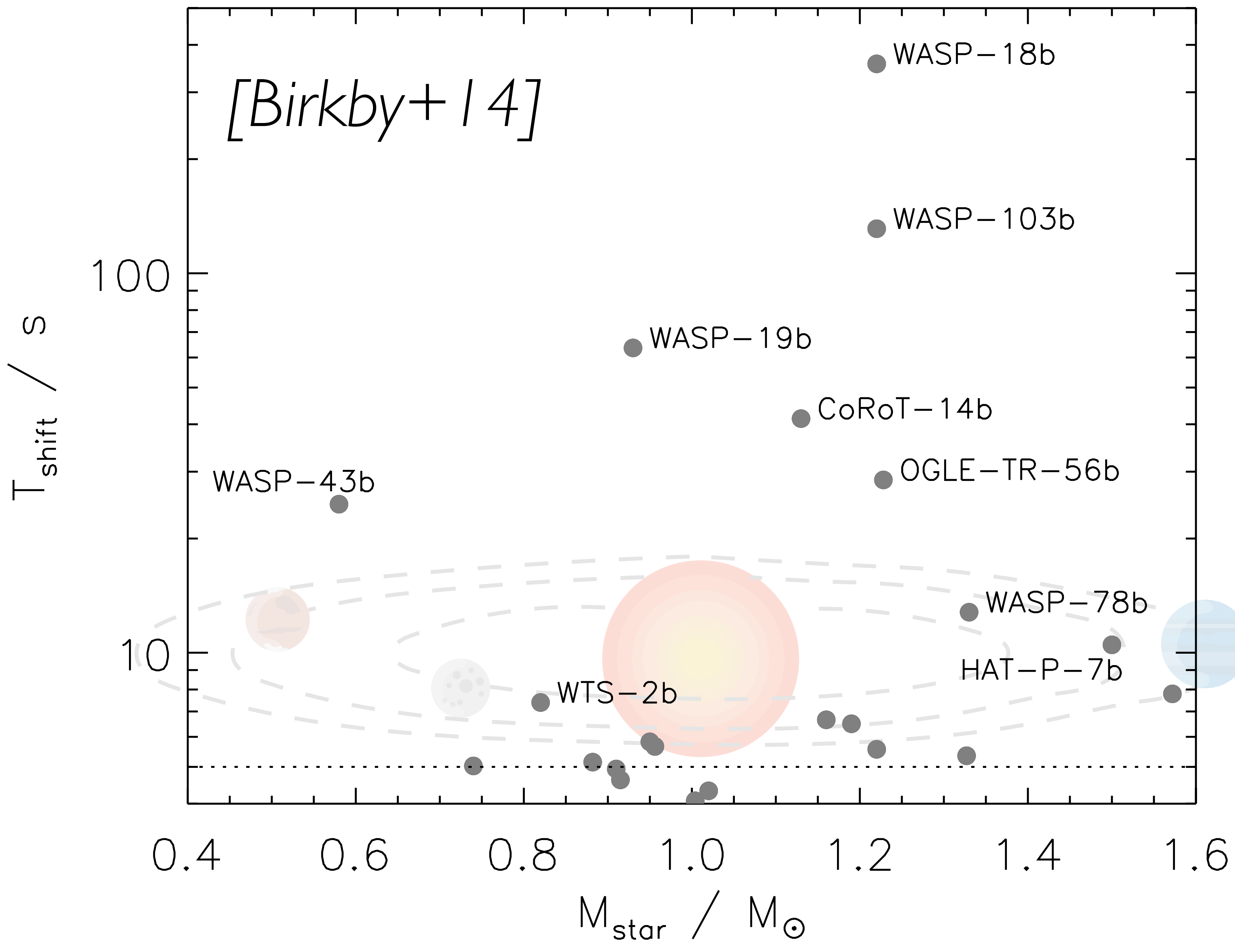
Observational constraints

- For planetary systems **close** to the edge of **tidal disruption**, it could be possible to measure the **transit timing variation** due to the inward migration. *Birkby+ [2014]* showed that a baseline of a few years is necessary

- Recently ***Yee+ [2020]*** used the **transit timing variation** of the WASP-12b system (29 ± 2 ms yr^{-1}) to estimate $Q'_\star = 1.8 \times 10^5$

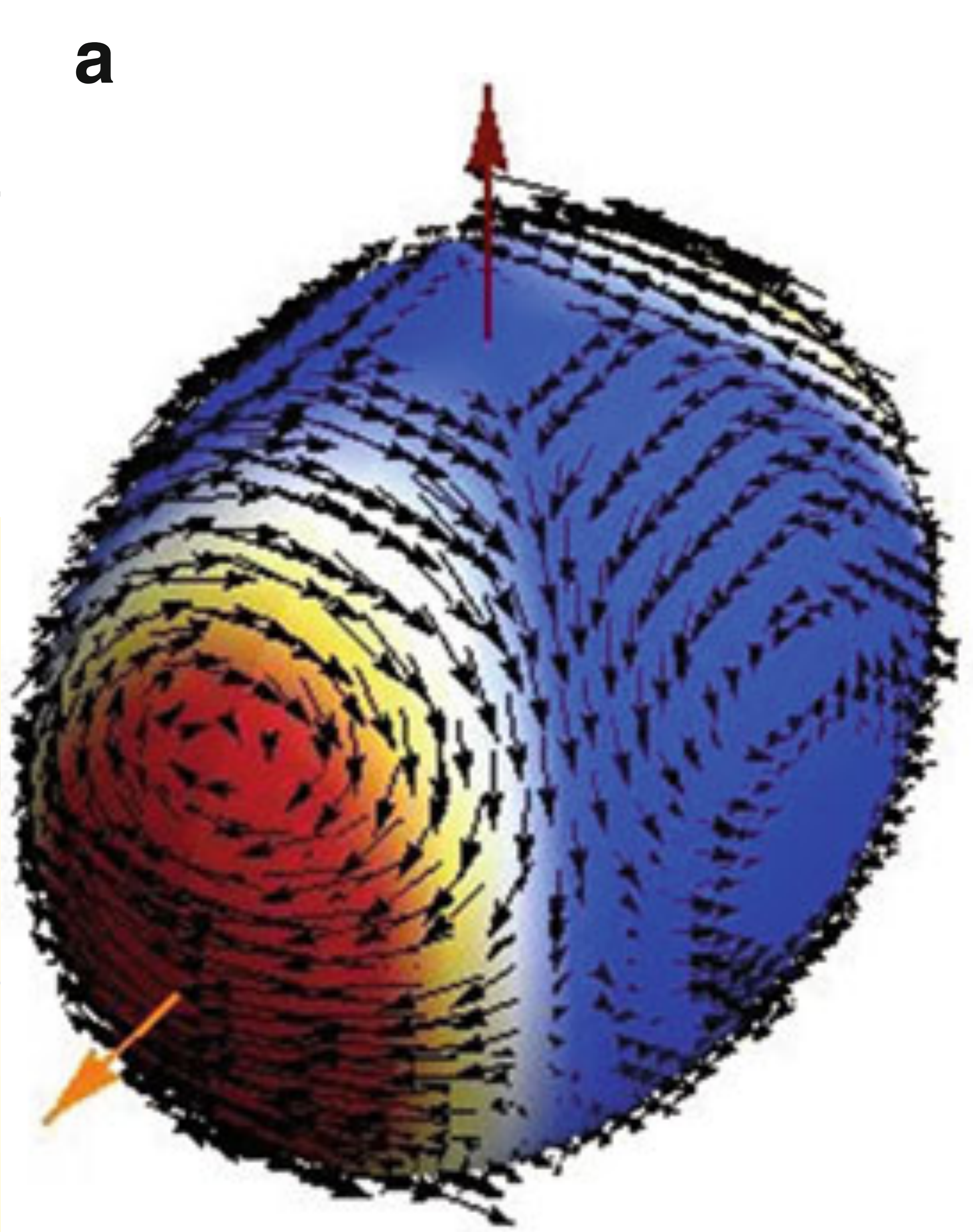


Transit time shift after 10 years assuming $Q'_\star = 10^6$ (circular orbits)



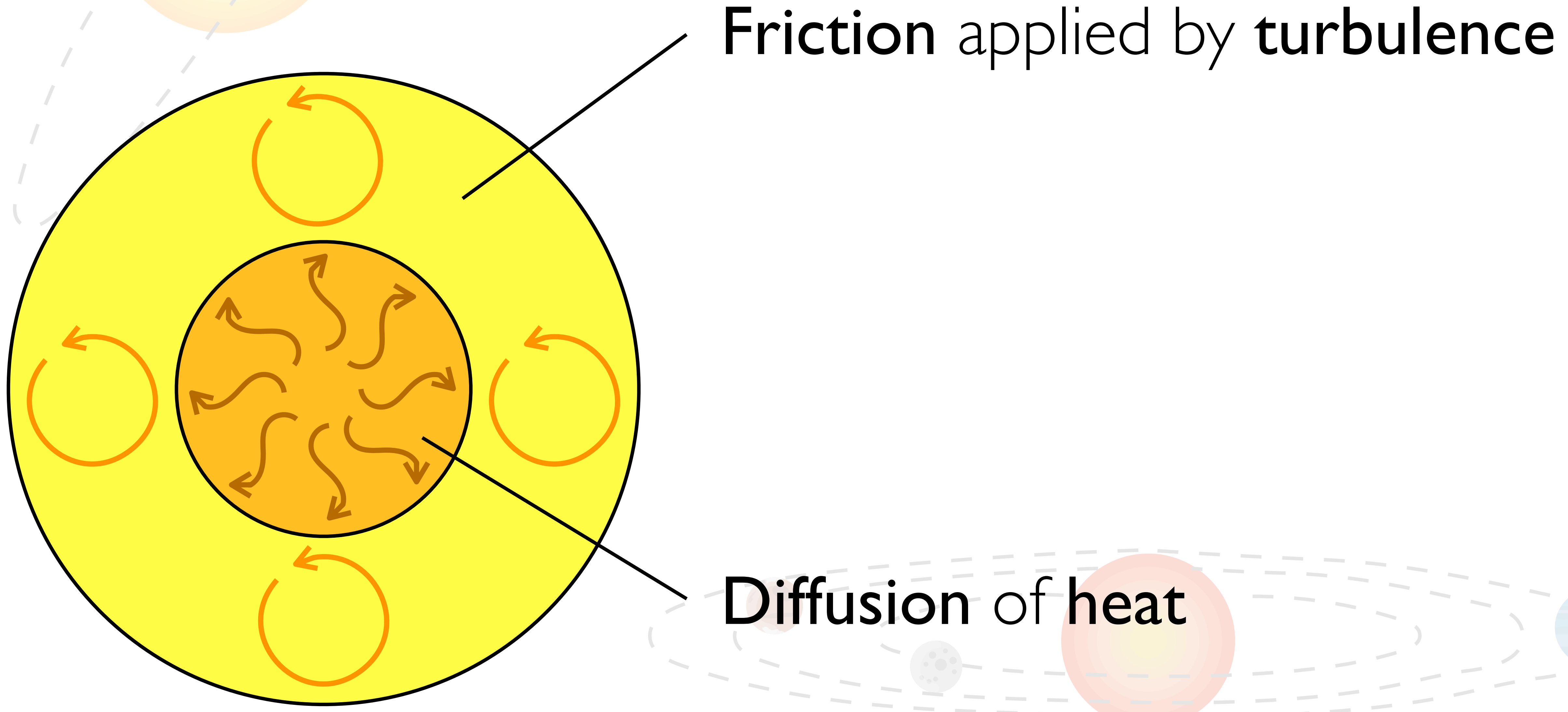
Stellar tide: equilibrium tide

Velocity field of the **equilibrium tide**



[Zahn 1966a; Remus+12]

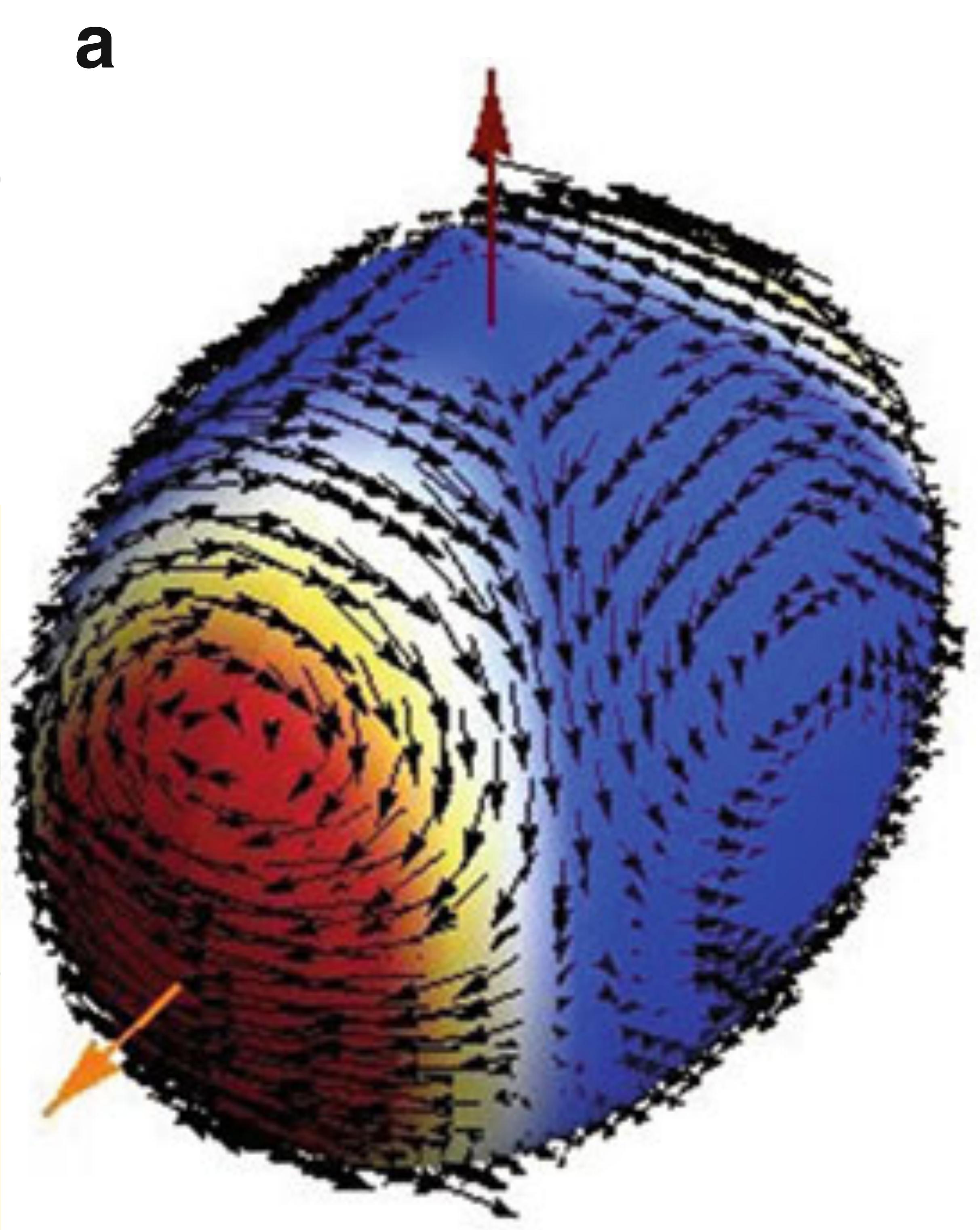
Dissipation



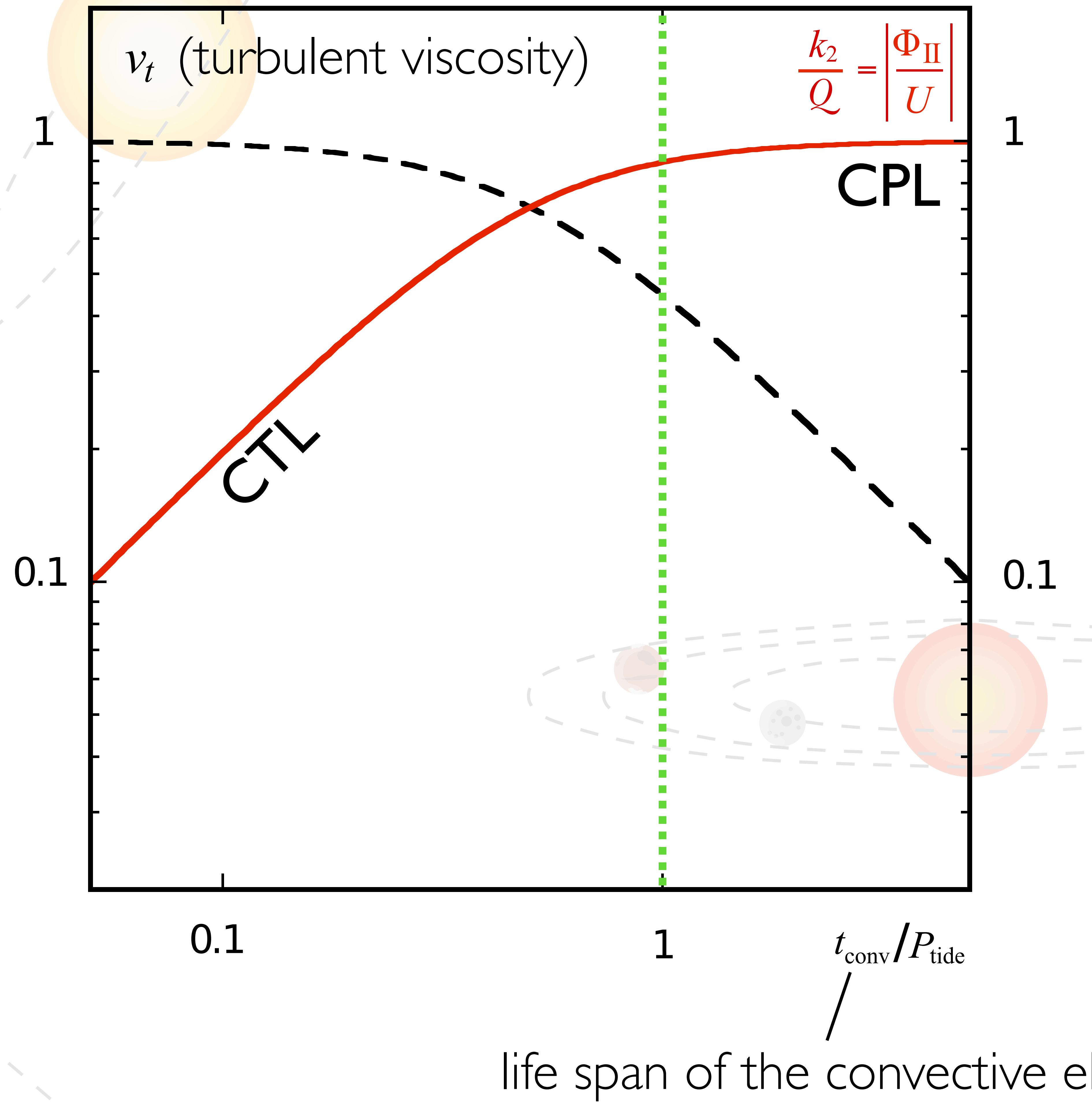
Dissipation in **convective region** is **higher** *[Zahn 1977]*

Stellar tide: equilibrium tide

Velocity field of the equilibrium tide

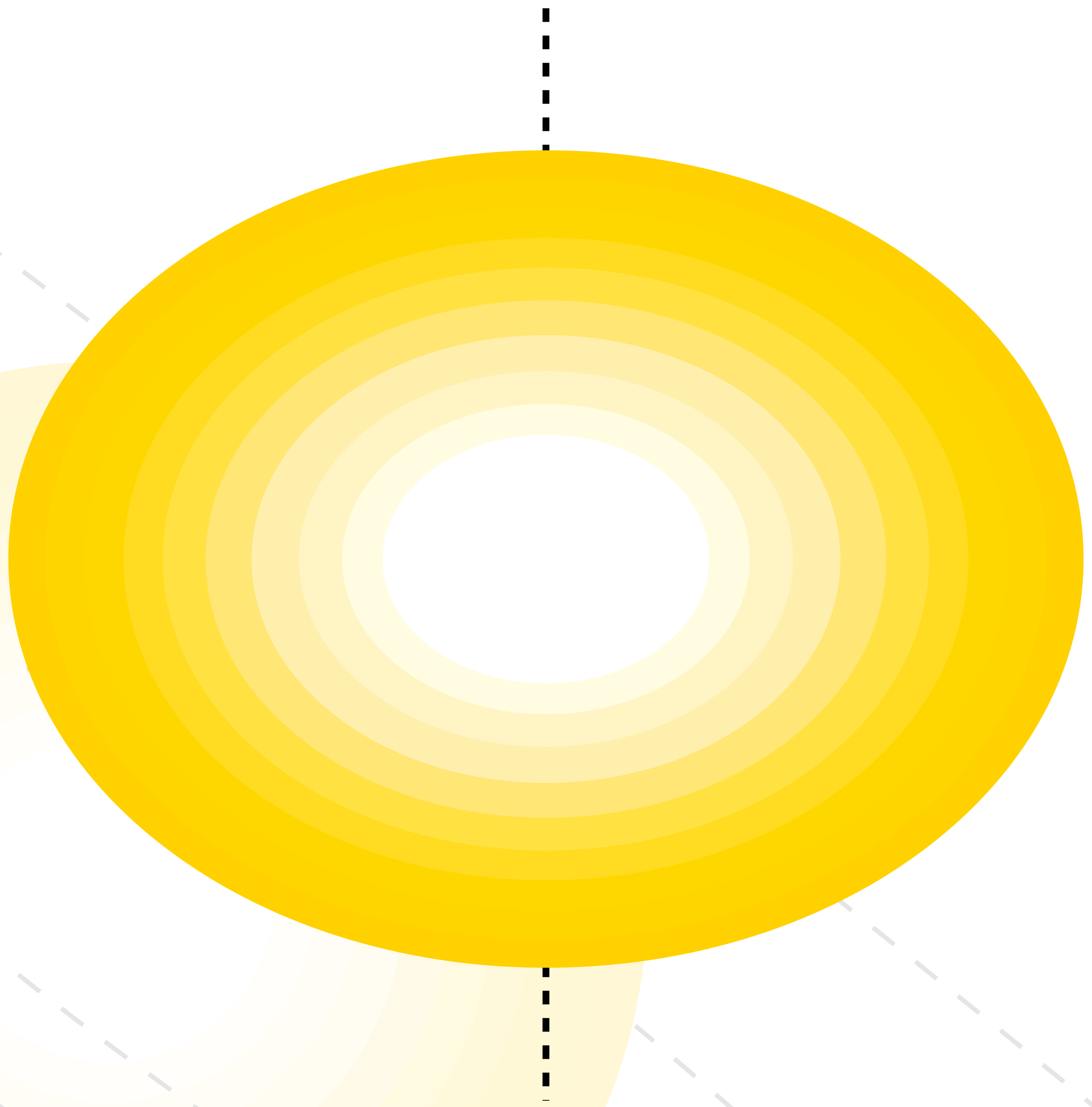


[Zahn 1966a; Remus+12]

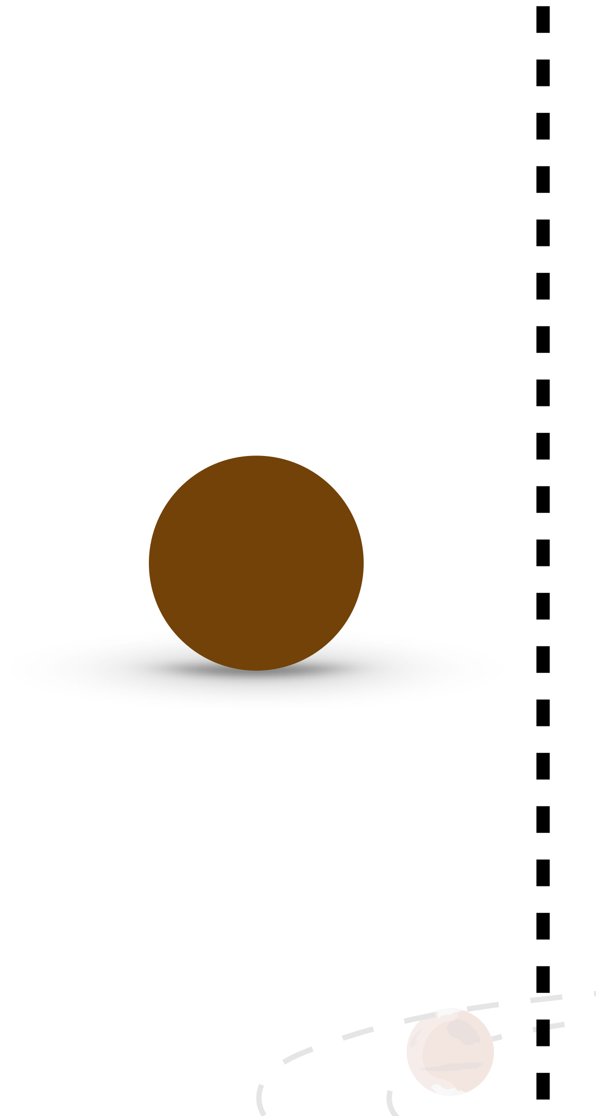


Stellar tide: dynamical tide

Tidal inertial waves in the **convective zone**



$P_{orb} = 1/2\ P_{\star}$



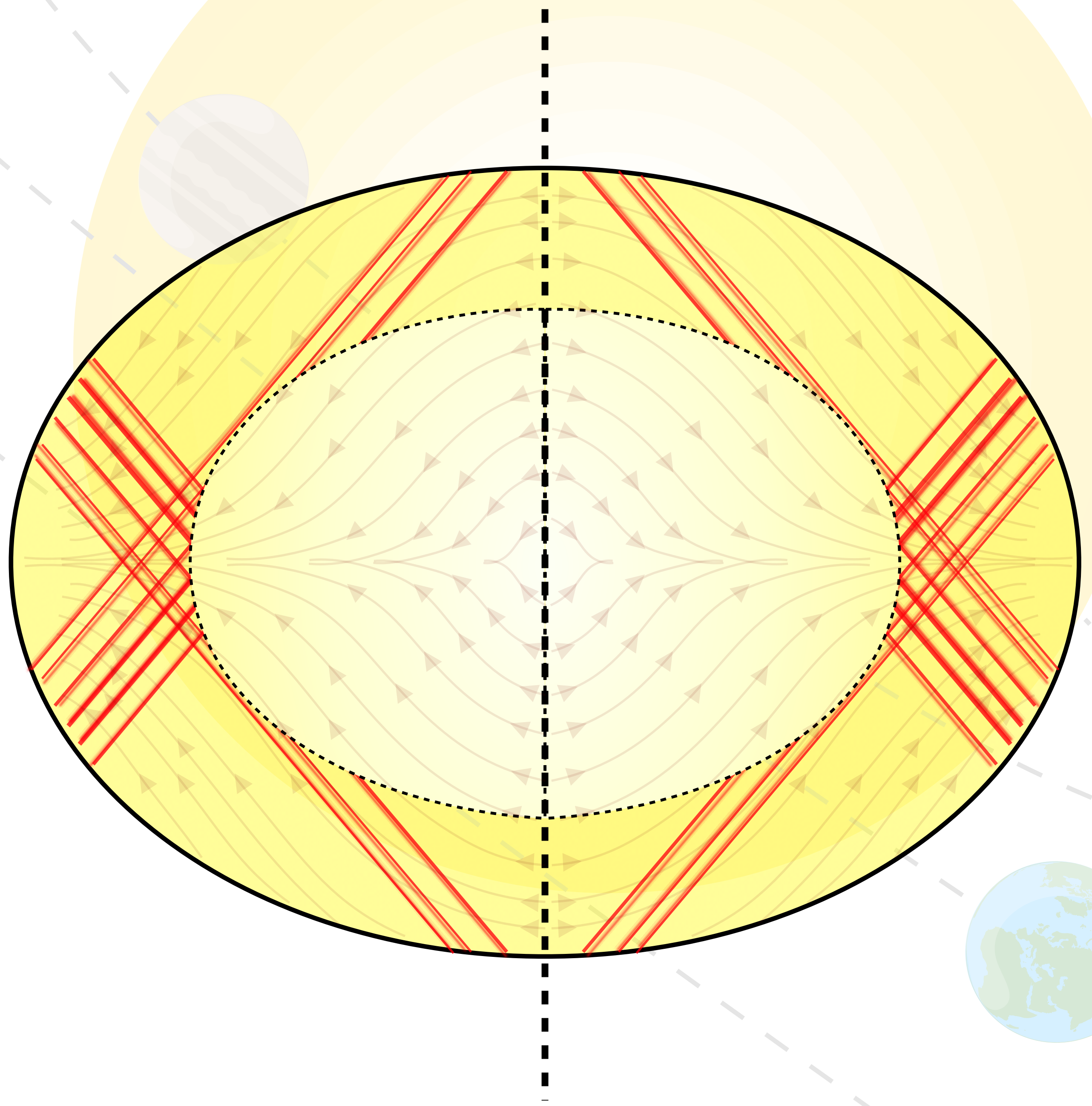
Equilibrium tide

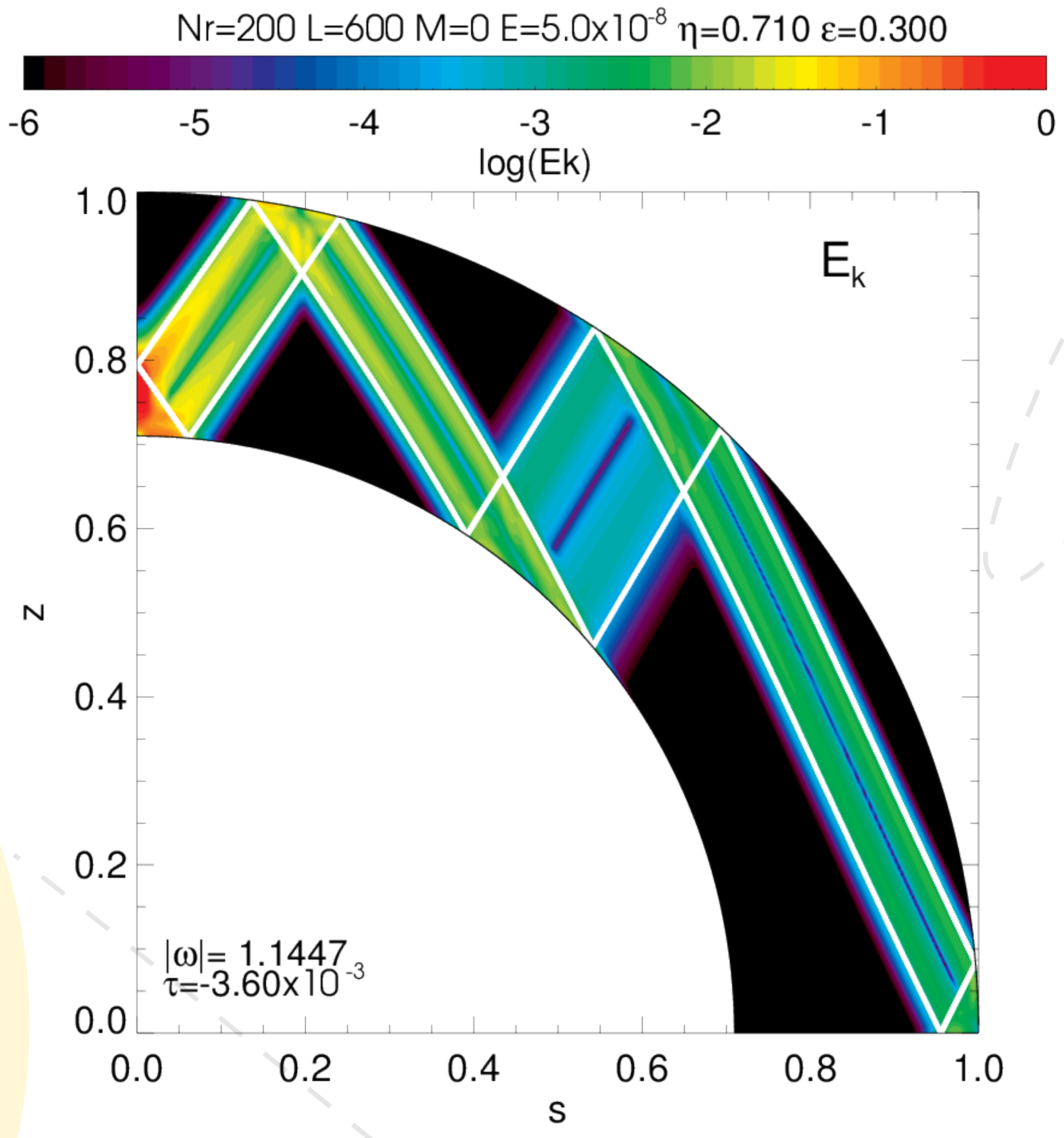
Equilibrium tide

Dynamical tide

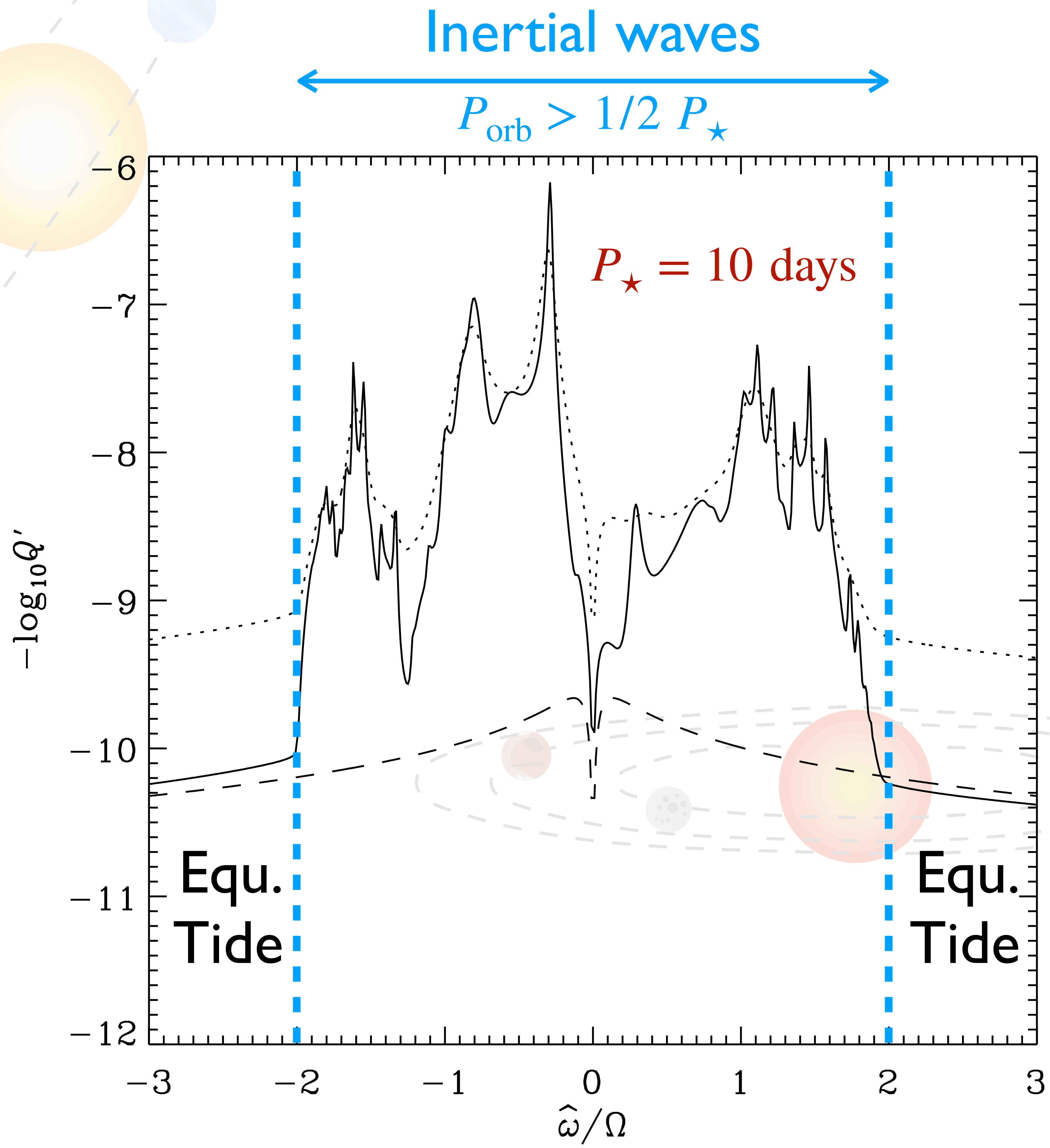
Stellar tide: dynamical tide

Tidal inertial waves in the convective zone





[Guenel+16]



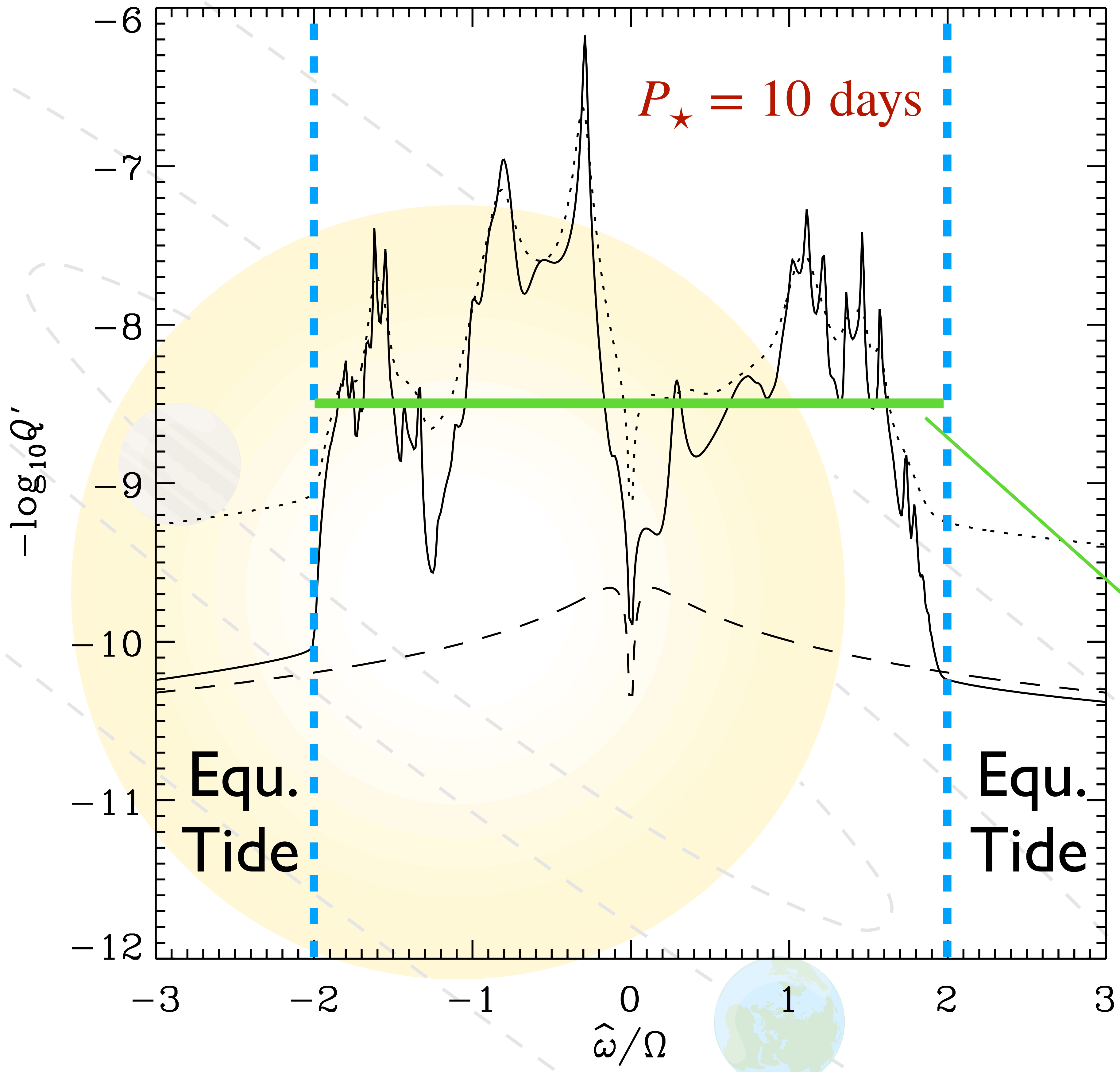
Sun, with a rotation period of 10 days *[Ogilvie & Lin, 07]*

Stellar tide: dynamical tide

Tidal **inertial waves** in the **convective zone**

Inertial waves

$P_{orb} > 1/2\ P_\star$



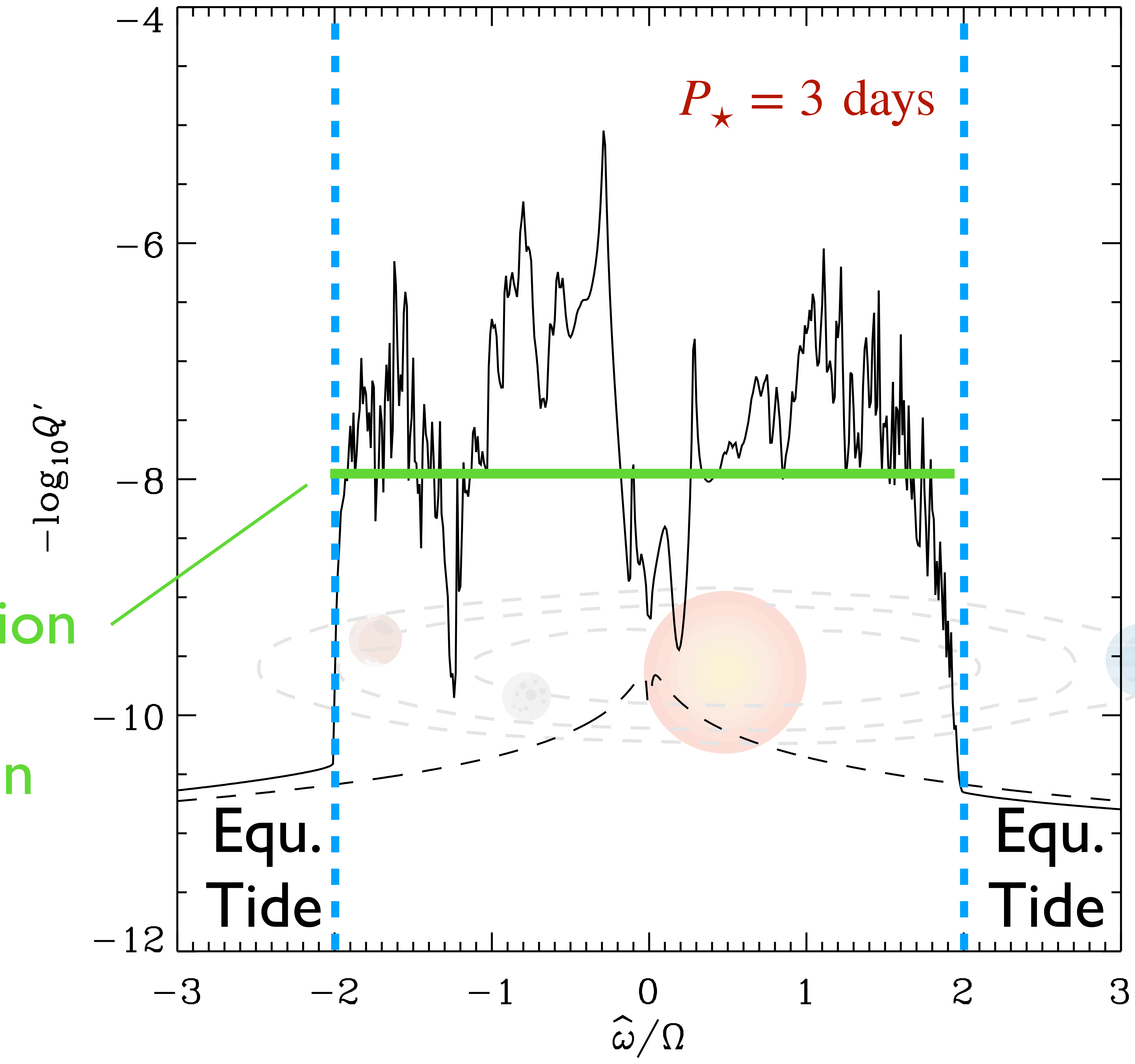
Sun, with a **rotation period** of **10 days** *[Ogilvie & Lin, 07]*

More peaks
Higher dissipation

Average value of **dissipation** depends on **structural parameters** and **rotation** *[Ogilvie 2013]*

Inertial waves

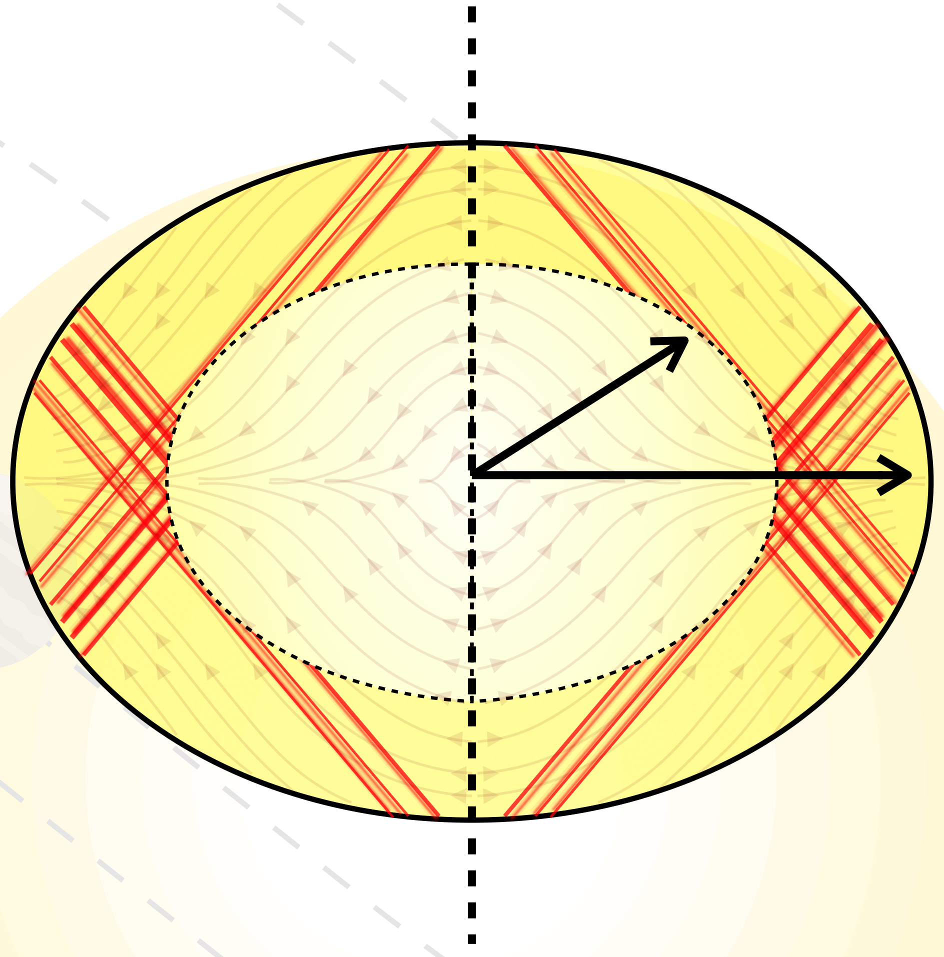
$P_{orb} > 1/2\ P_\star$



Sun, with a **rotation period** of **3 days** *[Ogilvie & Lin, 07]*

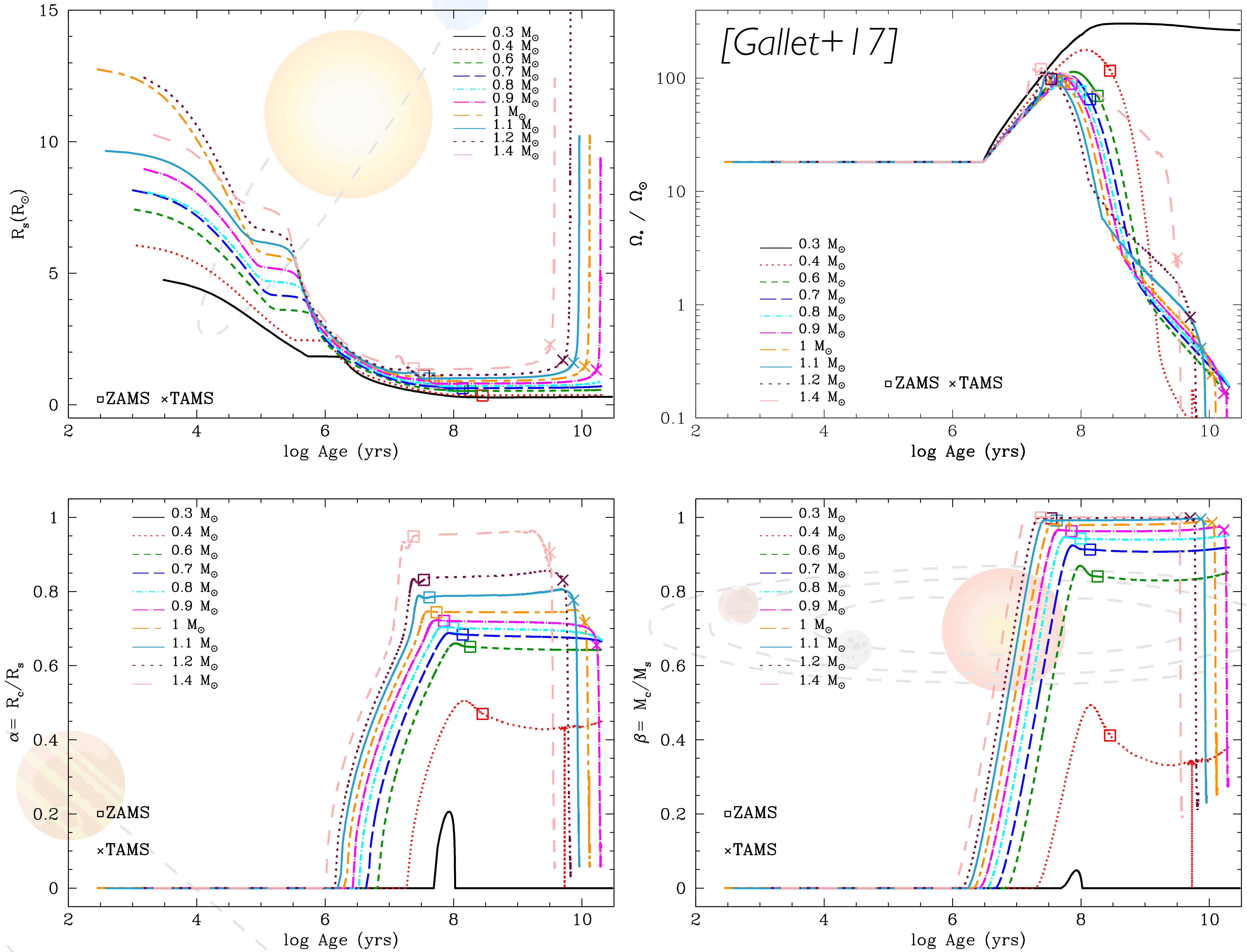
Stellar tide: dynamical tide

Tidal inertial waves in the **convective zone**



Average value of **dissipation** depends on **structural parameters** and **rotation**

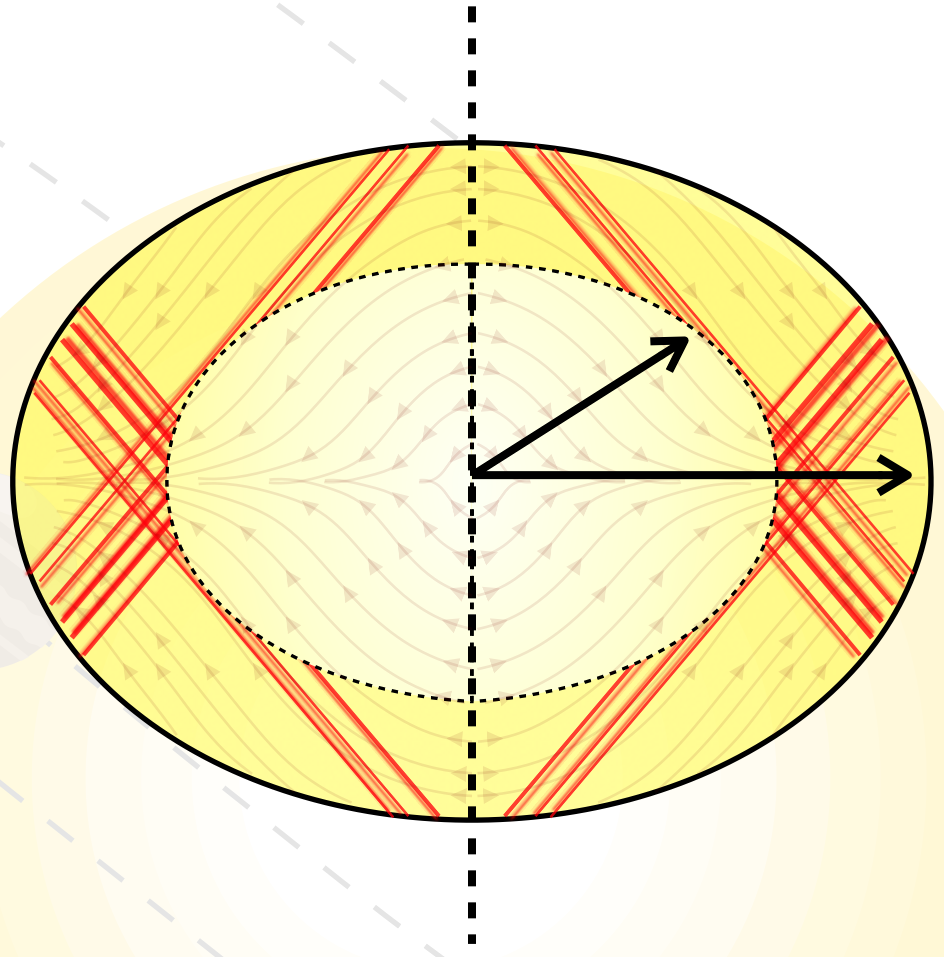
$\alpha = R_c/R_\star$
$\beta = M_c/M_\star$



Stellar tide: dynamical tide

Tidal inertial waves in the **convective zone**

Average value of **dissipation** depends on **structural parameters** and **rotation**

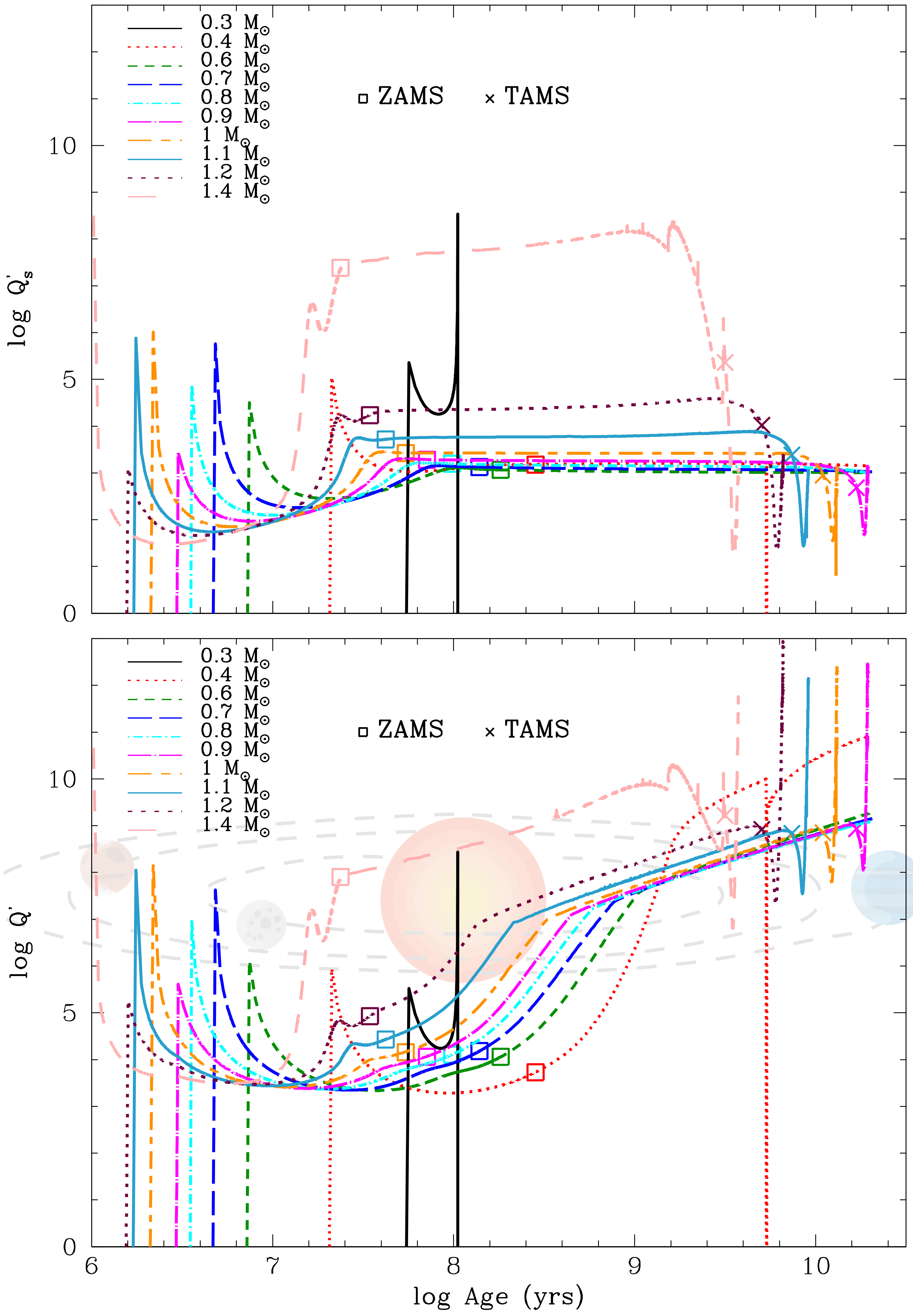


$$\alpha = R_c / R_\star$$
$$\beta = M_c / M_\star$$

$$\mathrm{Im} k_2 = f(\alpha, \beta) g(\Omega)$$

Only structural part ($\Omega = \mathrm{cst}$)

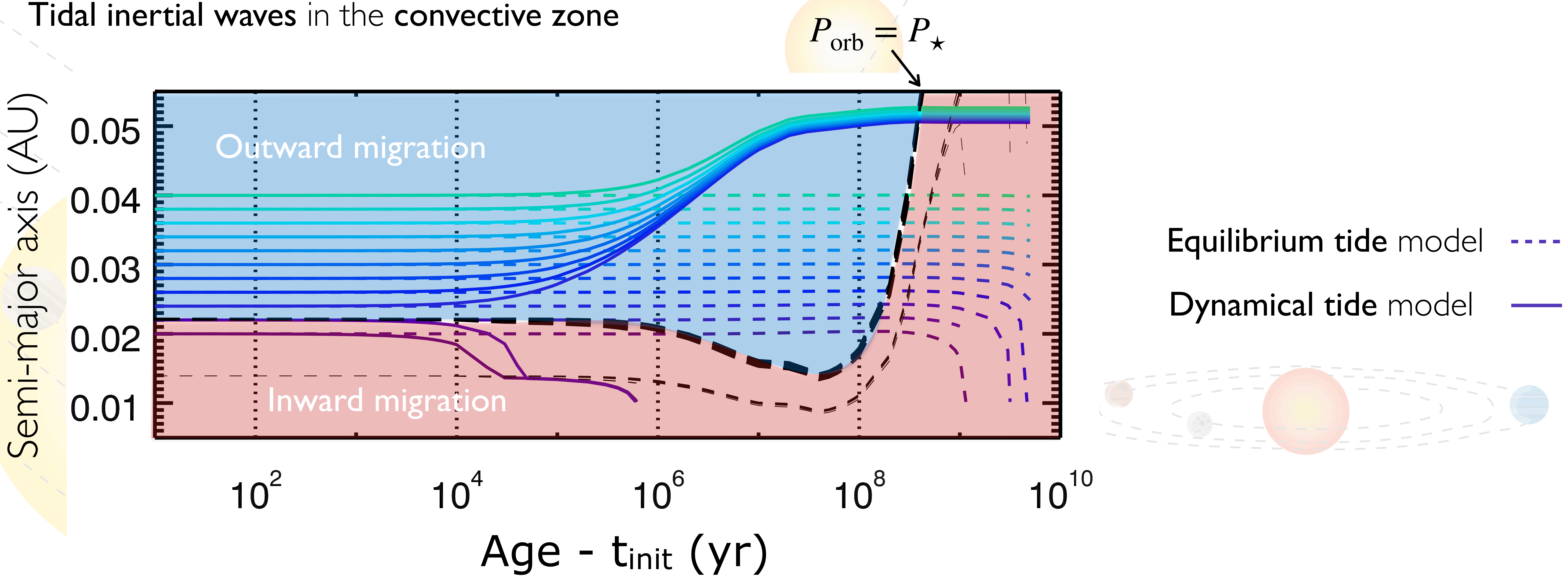
Structural part + rotational part



Stellar tide: dynamical tide

Tidal response → Orbital/rotational evolution

Tidal inertial waves in the **convective zone**

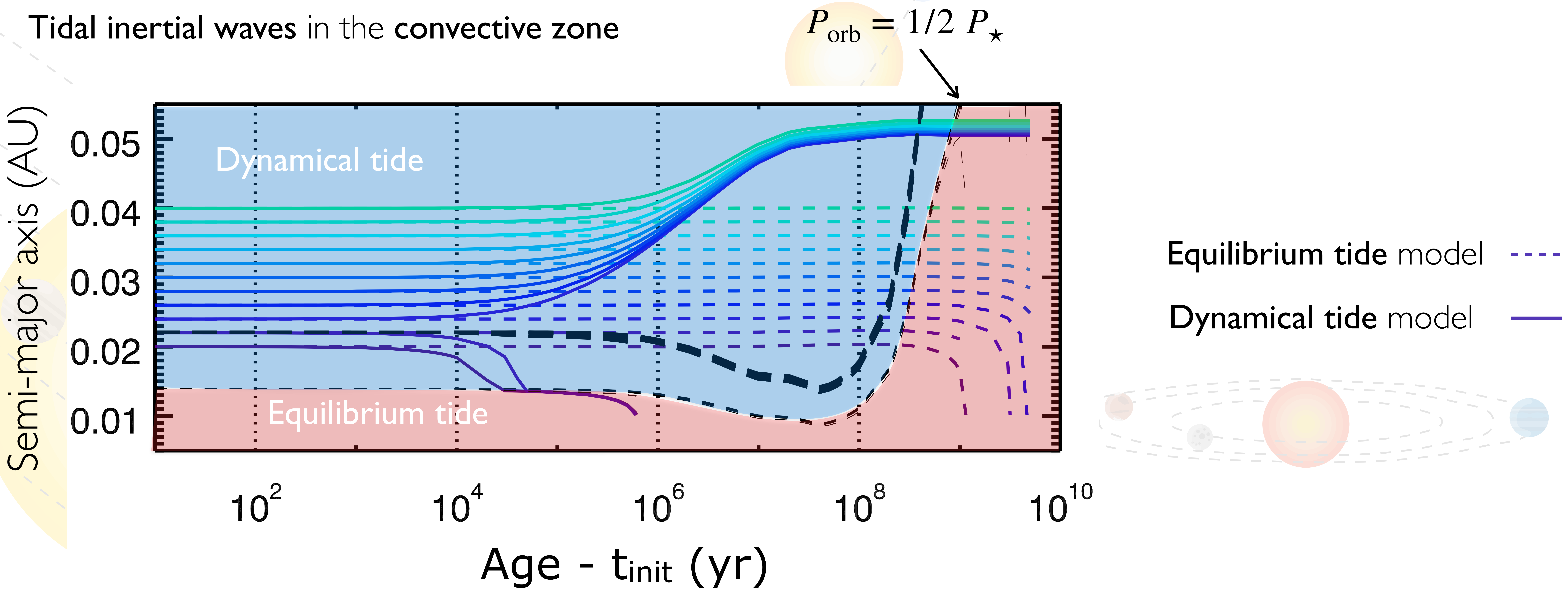


[Bolmont&Mathis16]

Stellar tide: dynamical tide

Tidal response → Orbital/rotational evolution

Tidal inertial waves in the convective zone



[Bolmont&Mathis16]

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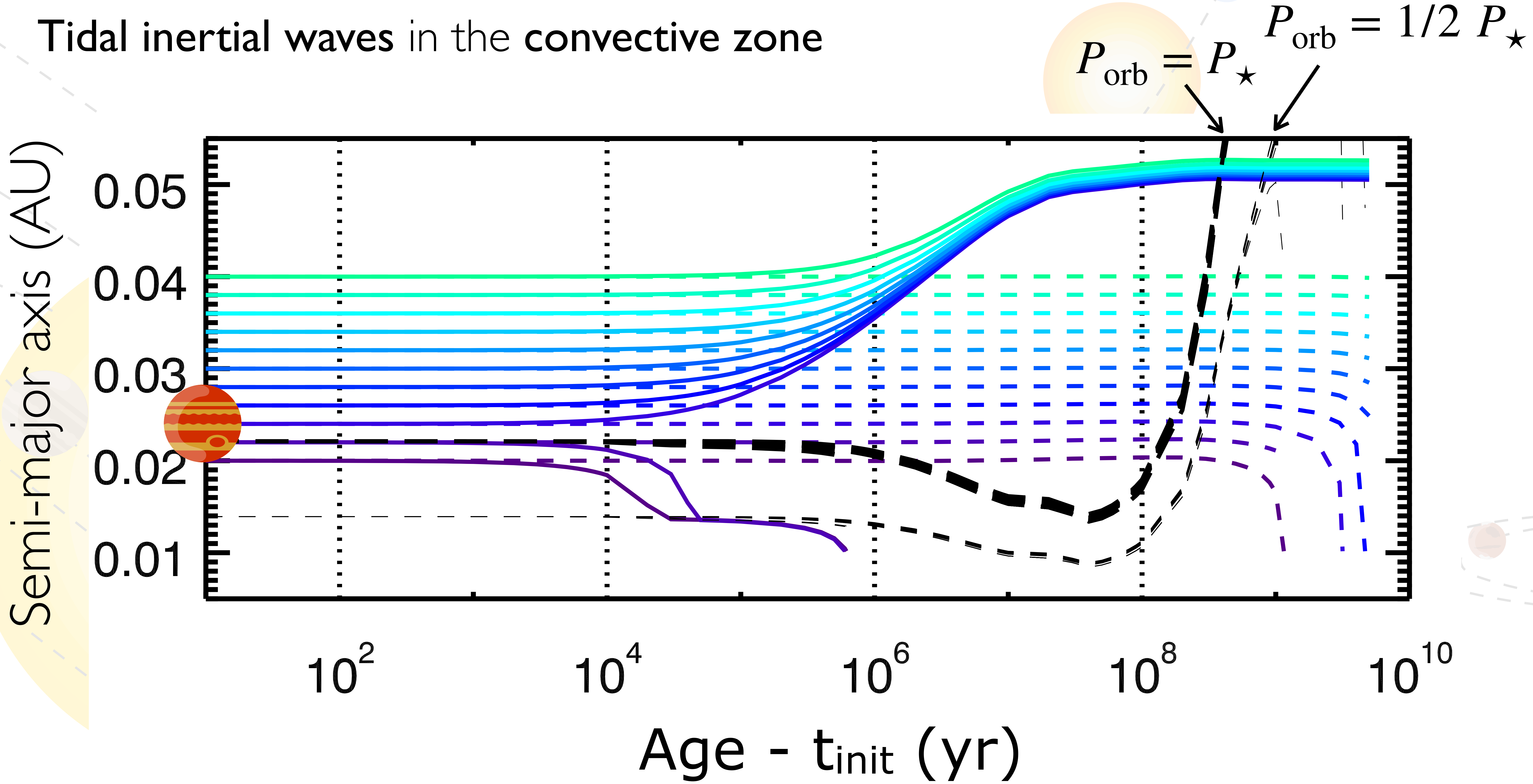
Tidal inertial waves in the **convective zone**



[Bolmont&Mathis16]

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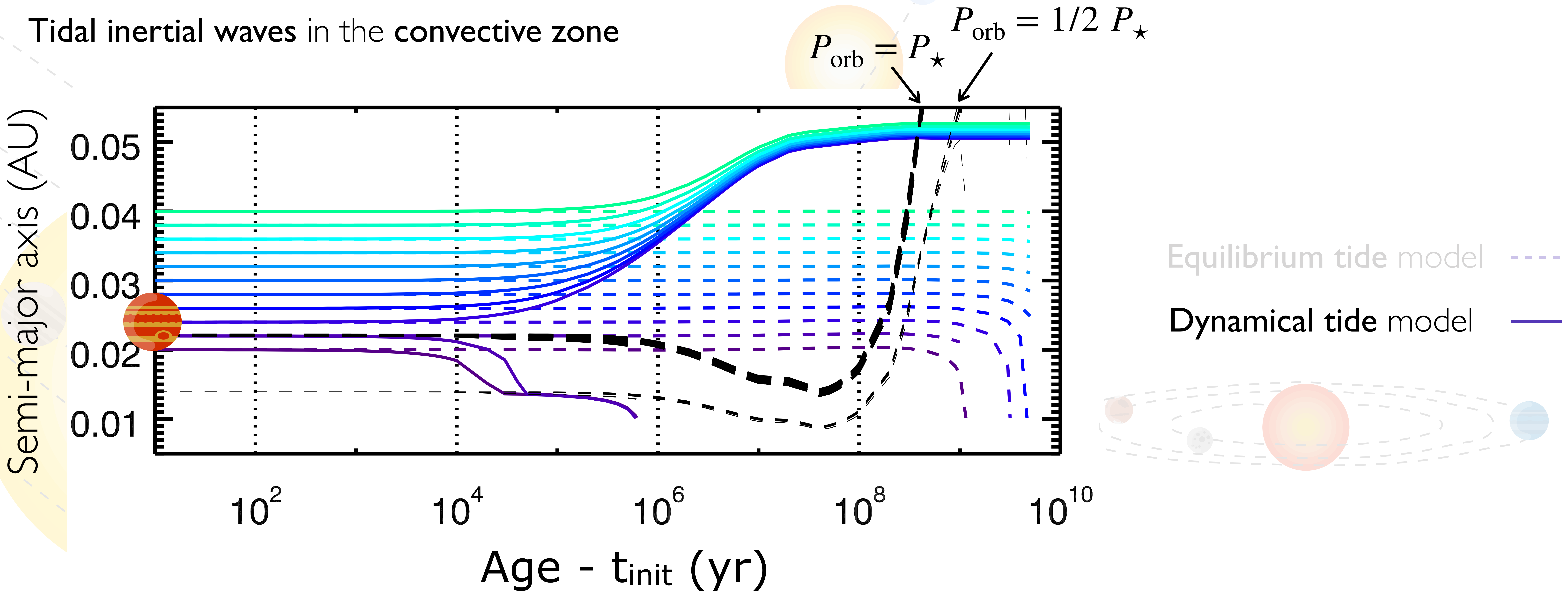
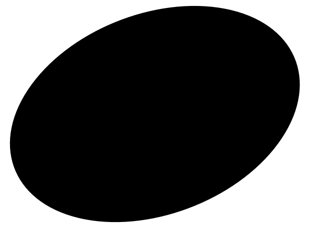
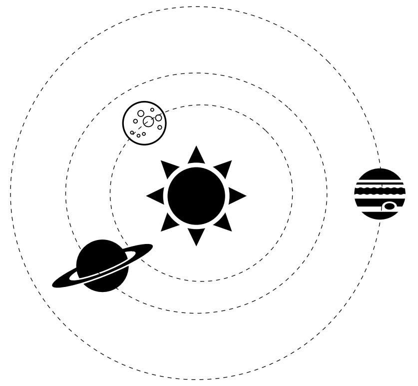
Tidal inertial waves in the convective zone



[Bolmont&Mathis16]

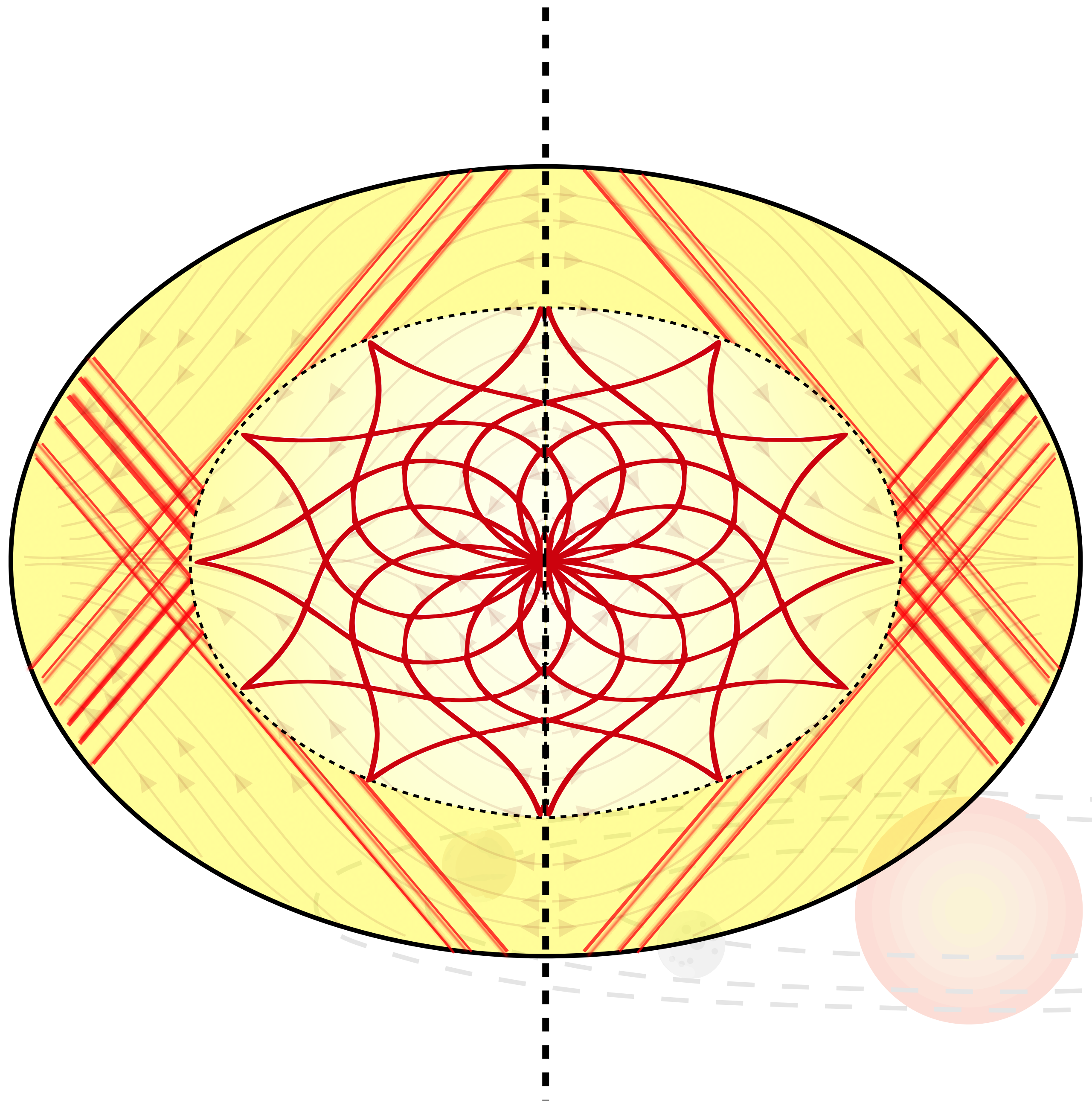
Stellar tide: dynamical tide

Tidal response  → Orbital/rotational evolution 

Effect of **tidal inertial waves** in the convective **envelope** of **Sun-like stars**:
Bolmont & Mathis 2016, Gallet+17, Benbakoura+19, Ahuir+21a…

➡ **Shapes** the **architecture** of the **young planetary systems**

Effect of **tidal gravity waves** in the radiative **zone** of **Sun-like stars**:
Ahuir+21b, Lazovik+21…



Stellar tide: dynamical tide

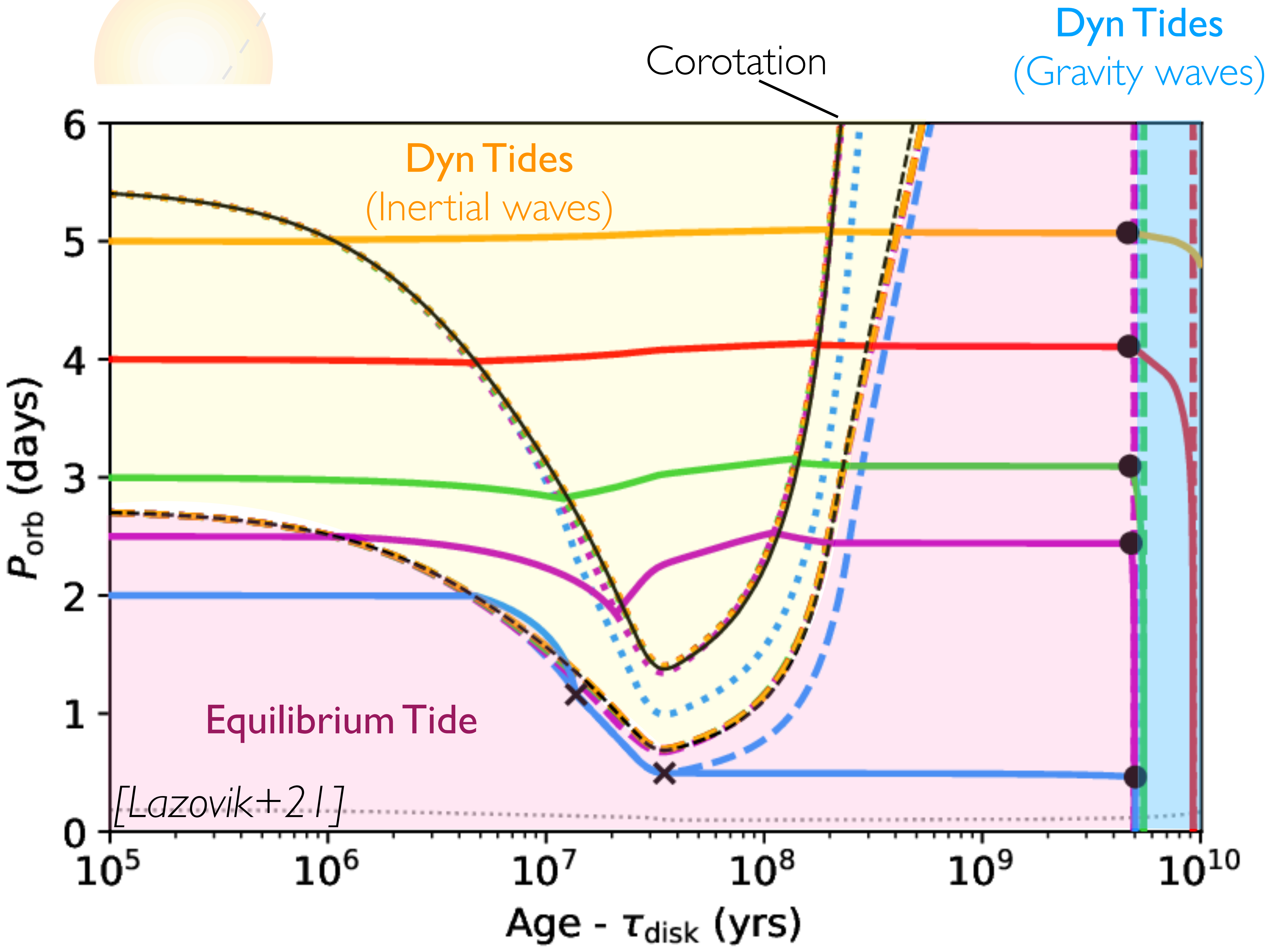
Tidal response → Orbital/rotational evolution

Effect of **tidal inertial waves** in the convective **envelope** of **Sun-like stars**:
Bolmont & Mathis 2016, Gallet+17, Benbakoura+19, Ahuir+21a…

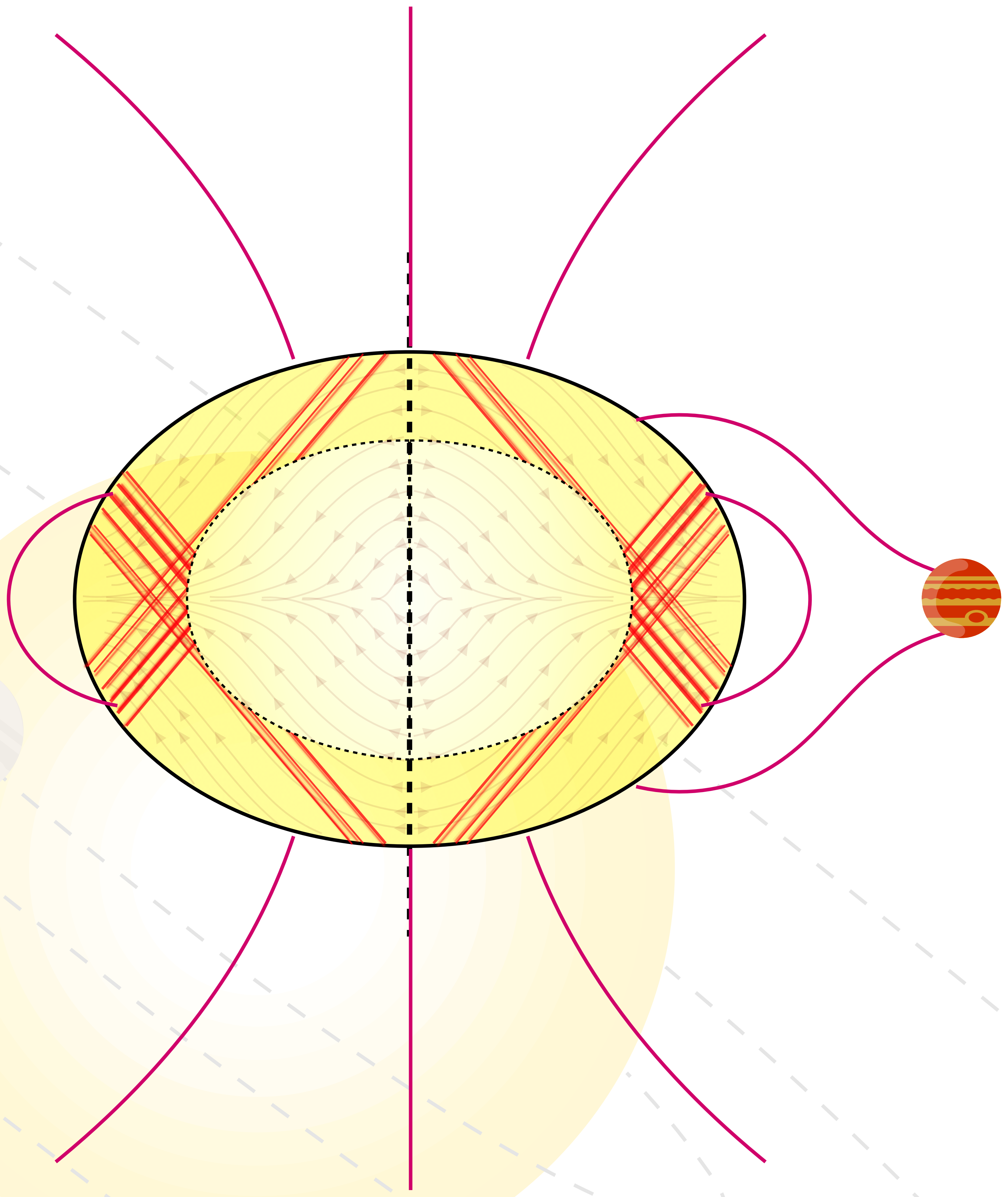
➡ **Shapes** the **architecture** of the young planetary systems

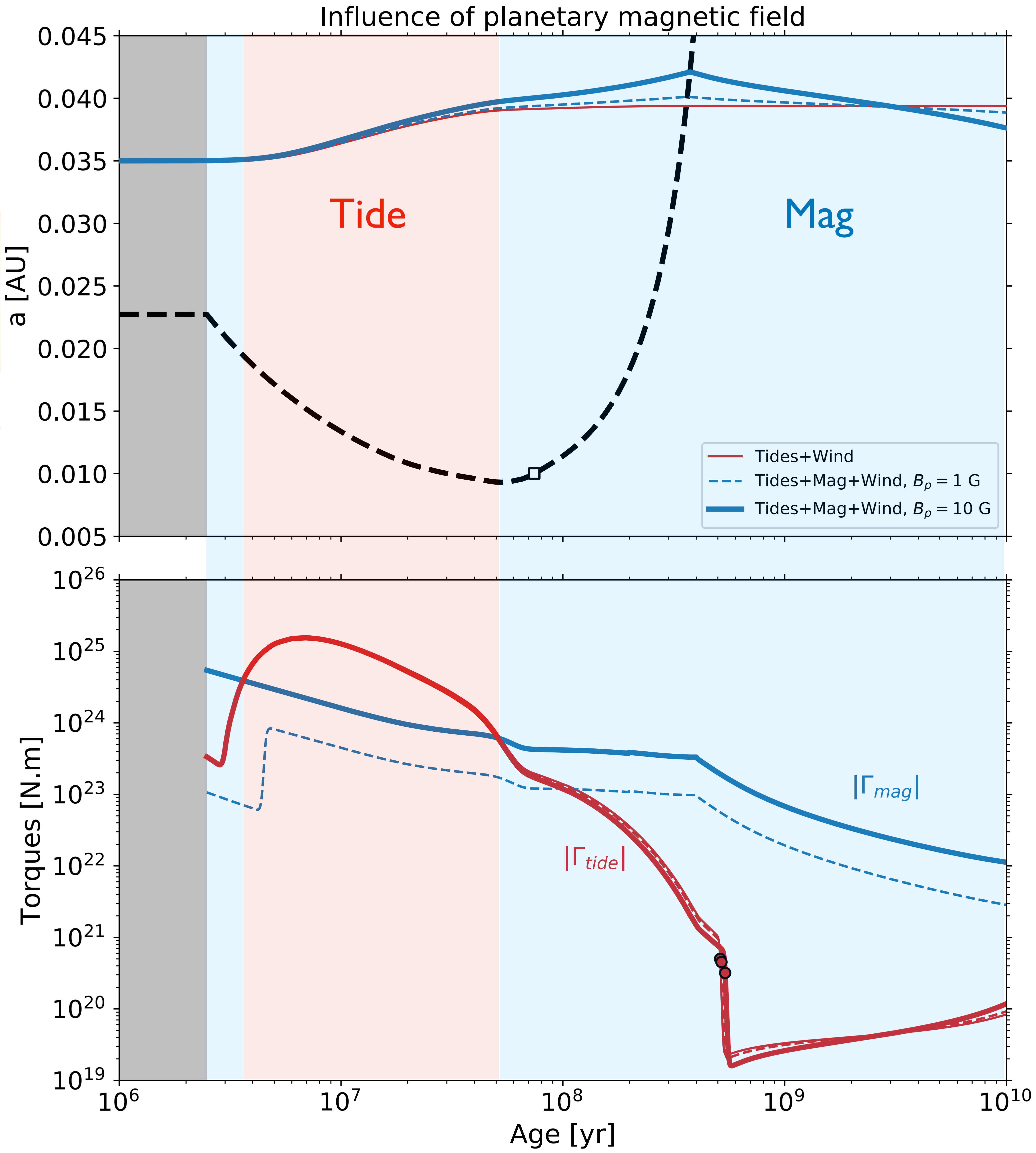
Effect of **tidal gravity waves** in the radiative **zone** of **Sun-like stars**:
Ahuir+21b, Lazovik+21…

➡ **Shapes** the **architecture** of the "old" planetary systems



Stellar tide: tide vs magnetism



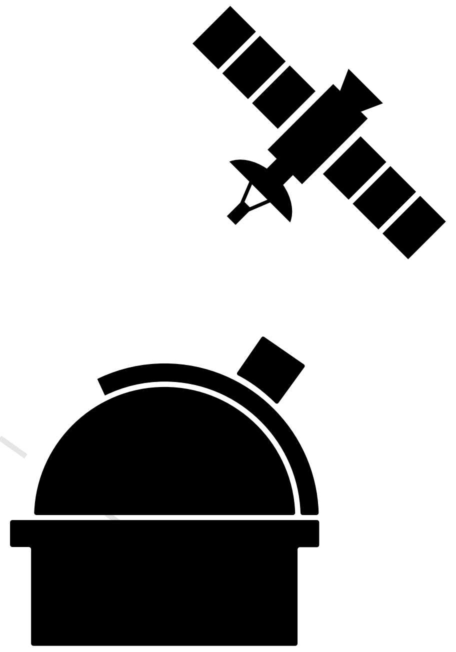


K star orbited by a magnetized hot Neptune *[Ahuir+21]*

Tidal interactions

- Why are tides important?
- A bit of theory
  - ★ Tides, the easy way
  - ★ Tides, the hard way…
- Some constraints brought by studying tides…
  - ★ Stars
  - ★ Planets
  - ★ Multi-planet systems

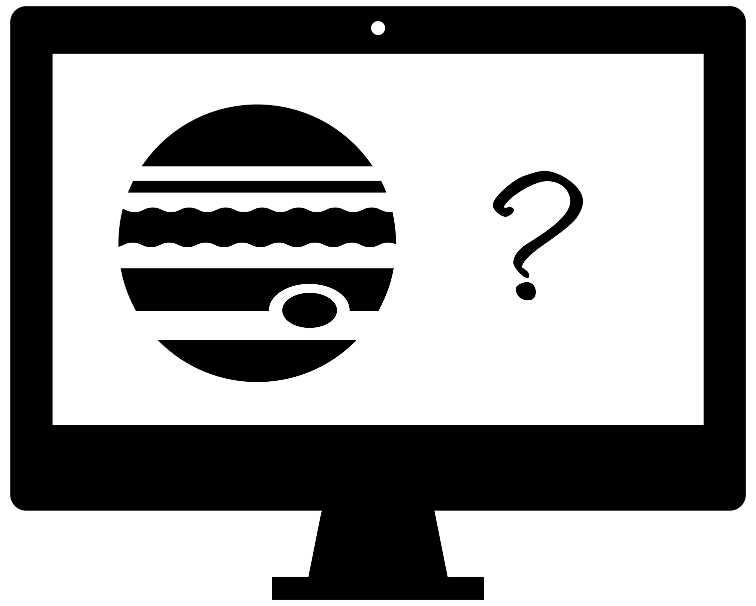
Planetary tide



Radius
Mass

→

Potential composition and internal structure *[Sotin+07]*

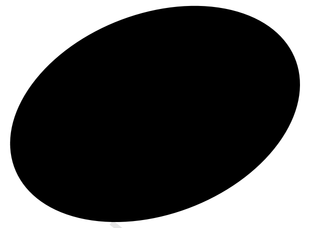


↓

Rheology

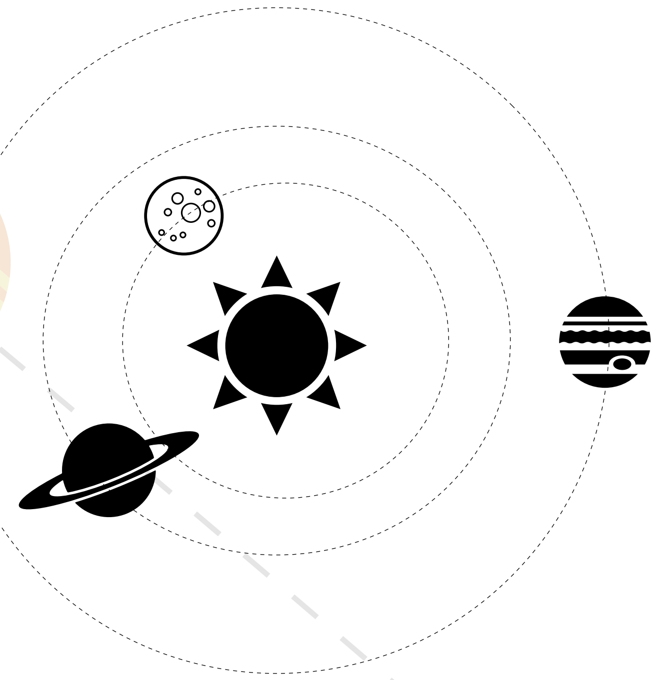
←

Tidal response



↓

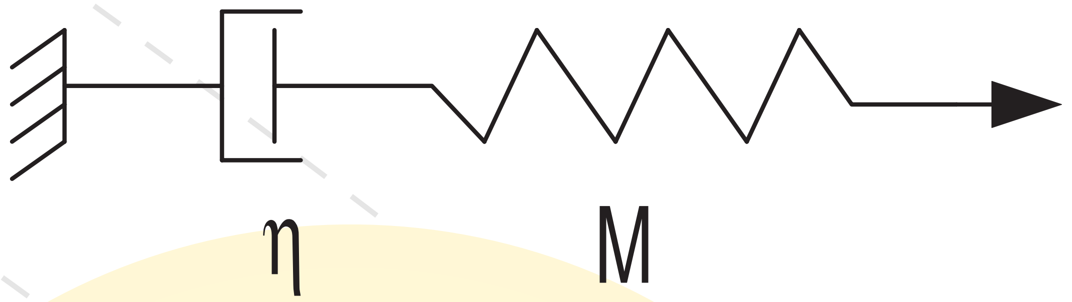
Orbital/rotational evolution



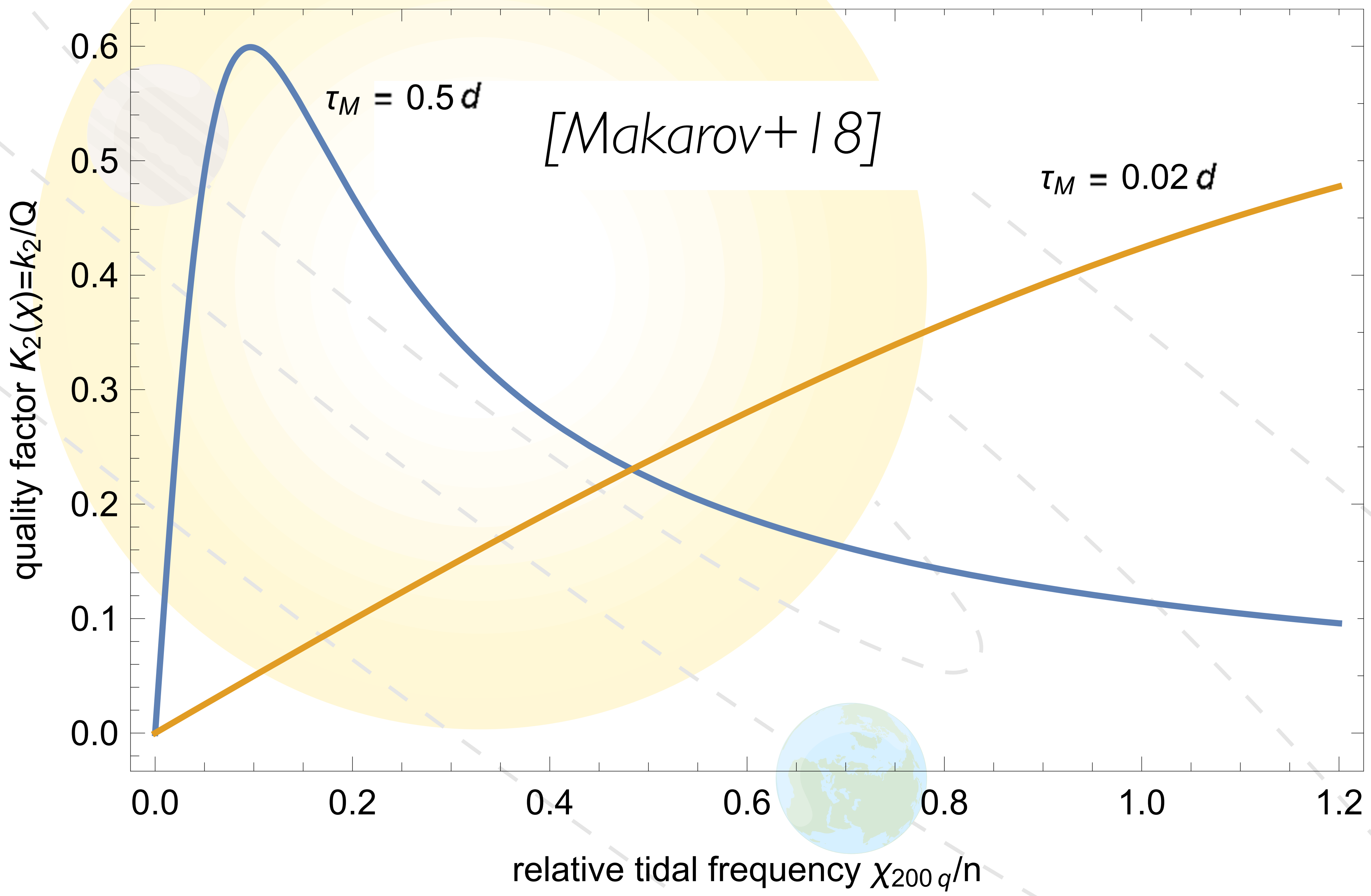
Planetary tide: rocky planets/cores

Rheology

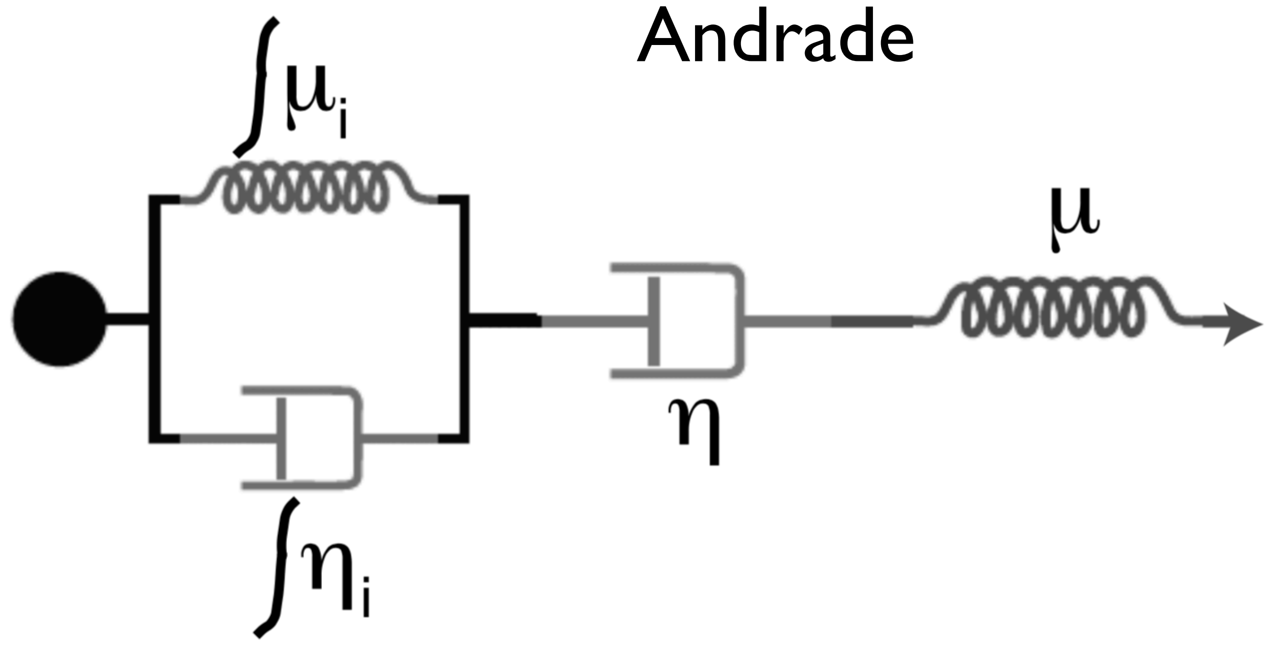
Tidal response

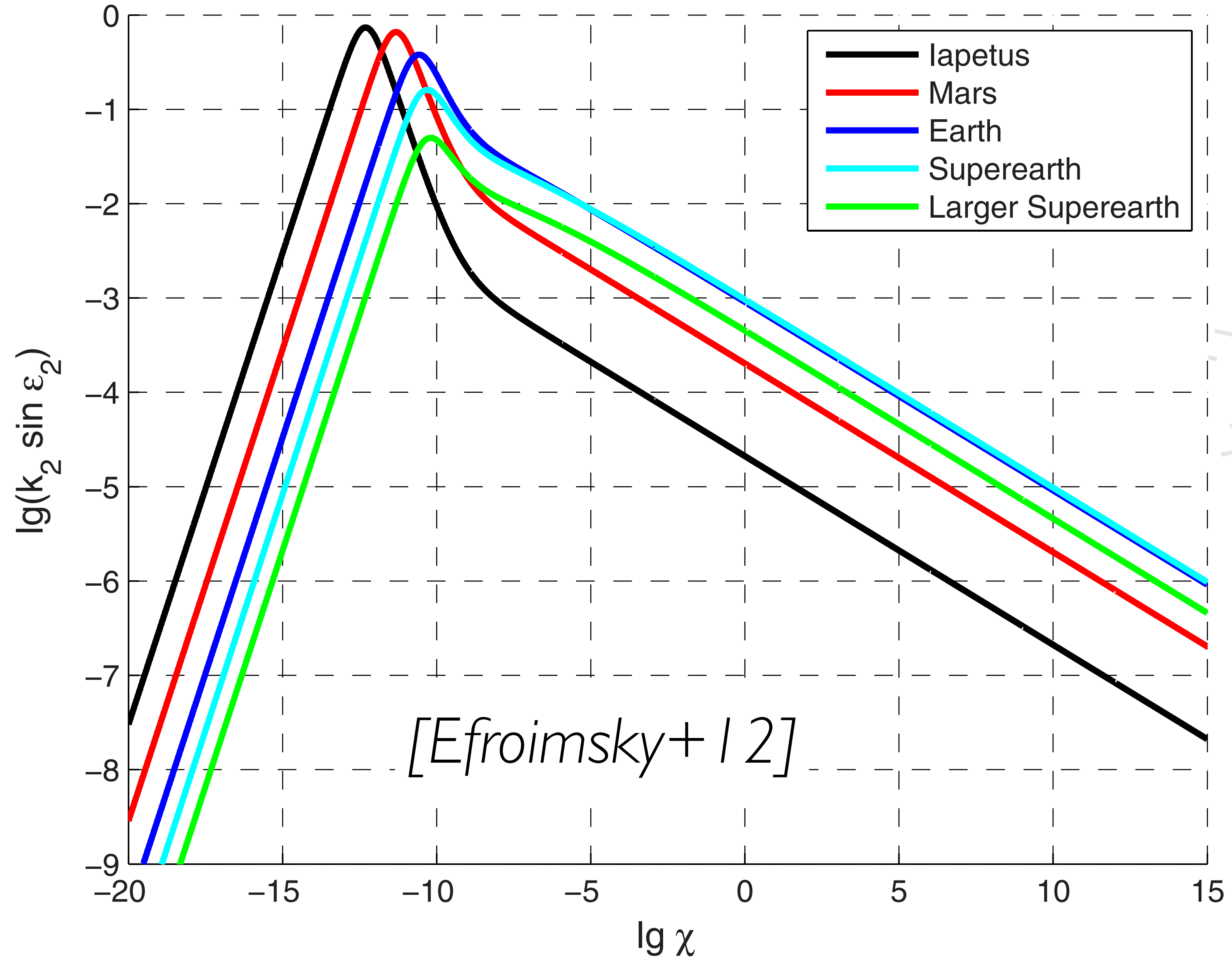


Maxwell

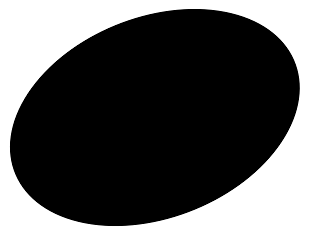
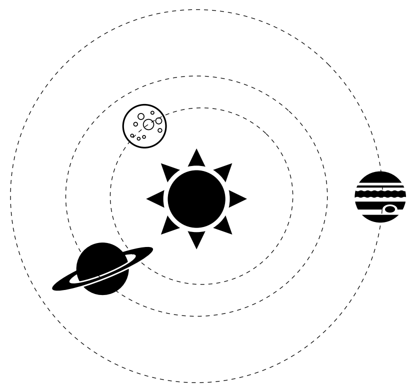


Andrade





Planetary tide: rocky planets/cores

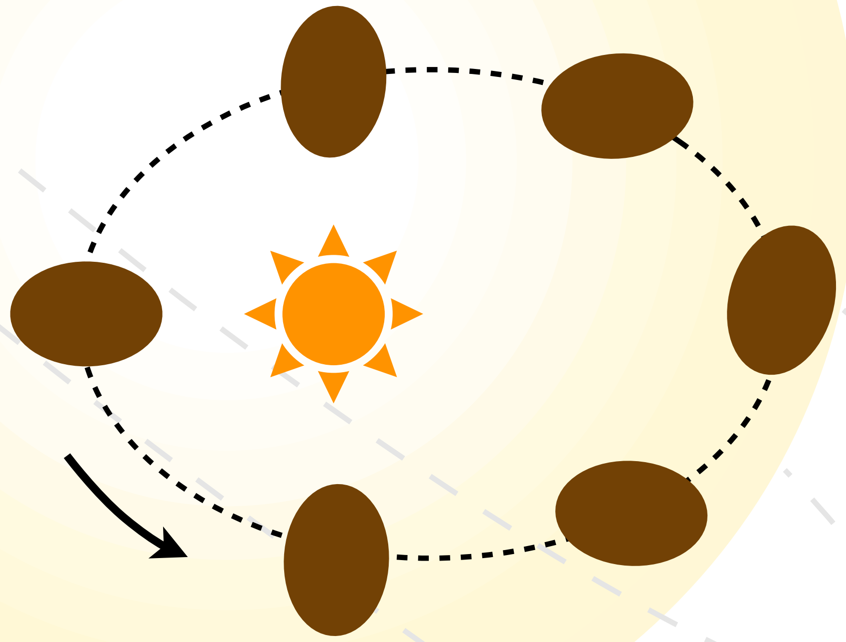
Tidal response  → Orbital/rotational evolution 

Weakly viscous fluid approximation

e.g. constant time lag model *[e.g. Hut 1981]*

Eccentricity = 0 ➜ Synchronization

Eccentricity ≠ 0 ➜ Pseudo-synchronization



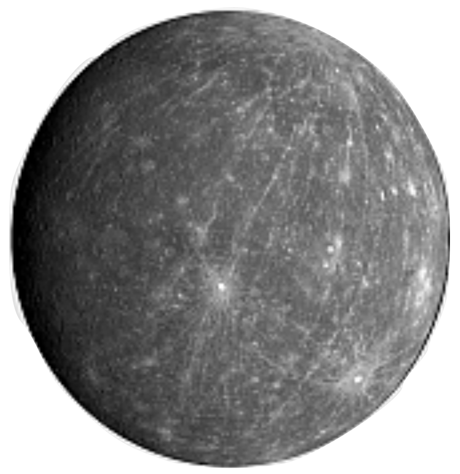
synchronization
around periastron

Anelastic material approximation

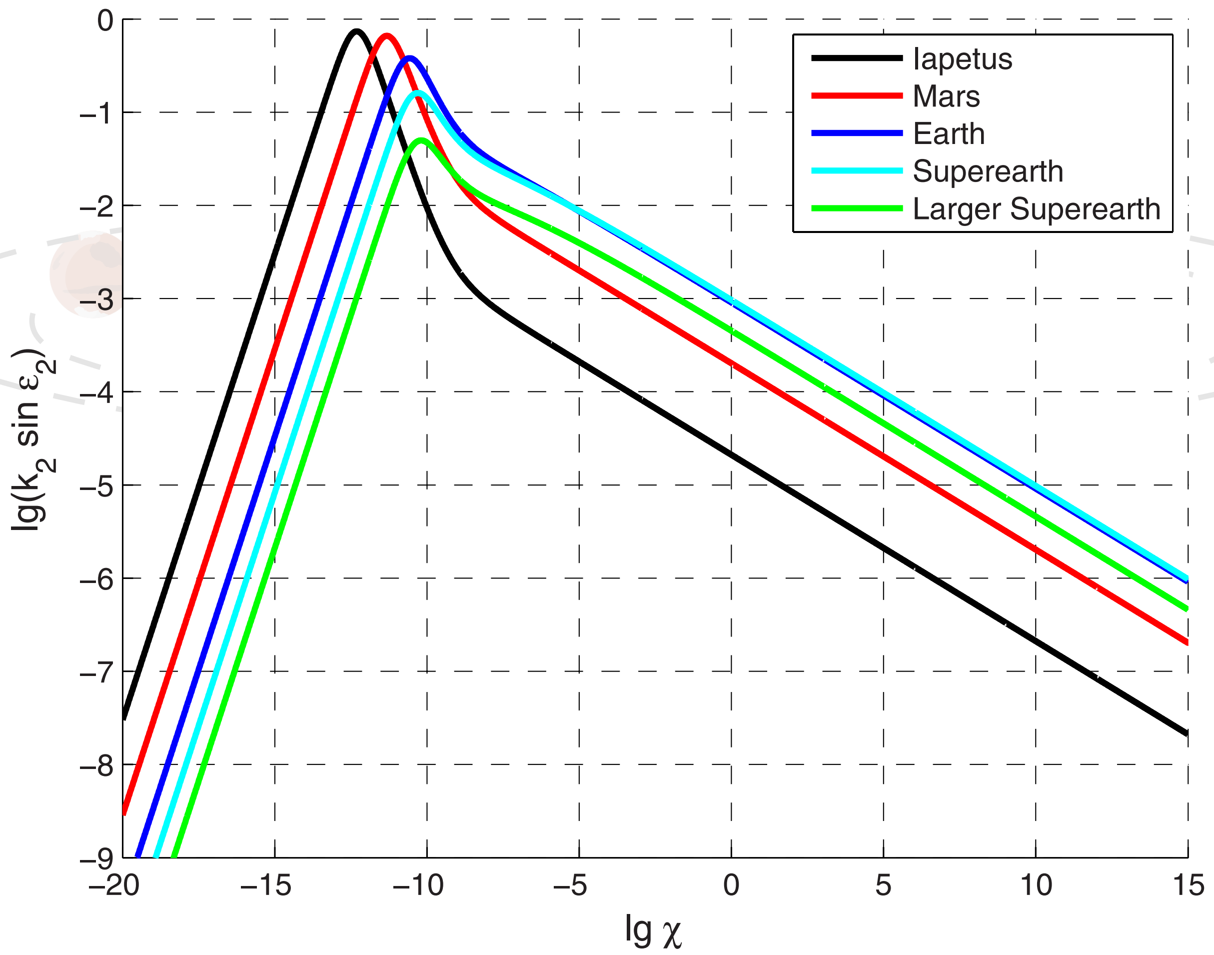
e.g. Andrade rheology *[e.g. Efroimsky+, Makarov+13]*

Eccentricity = 0 ➜ Synchronization

Eccentricity ≠ 0 ➜ Spin-orbit resonance

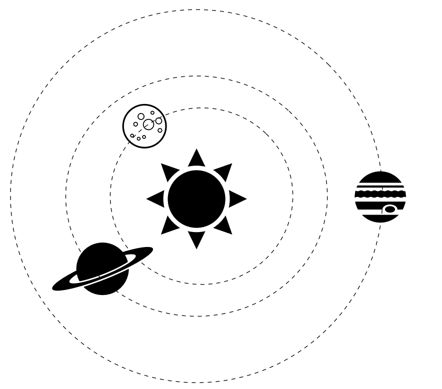


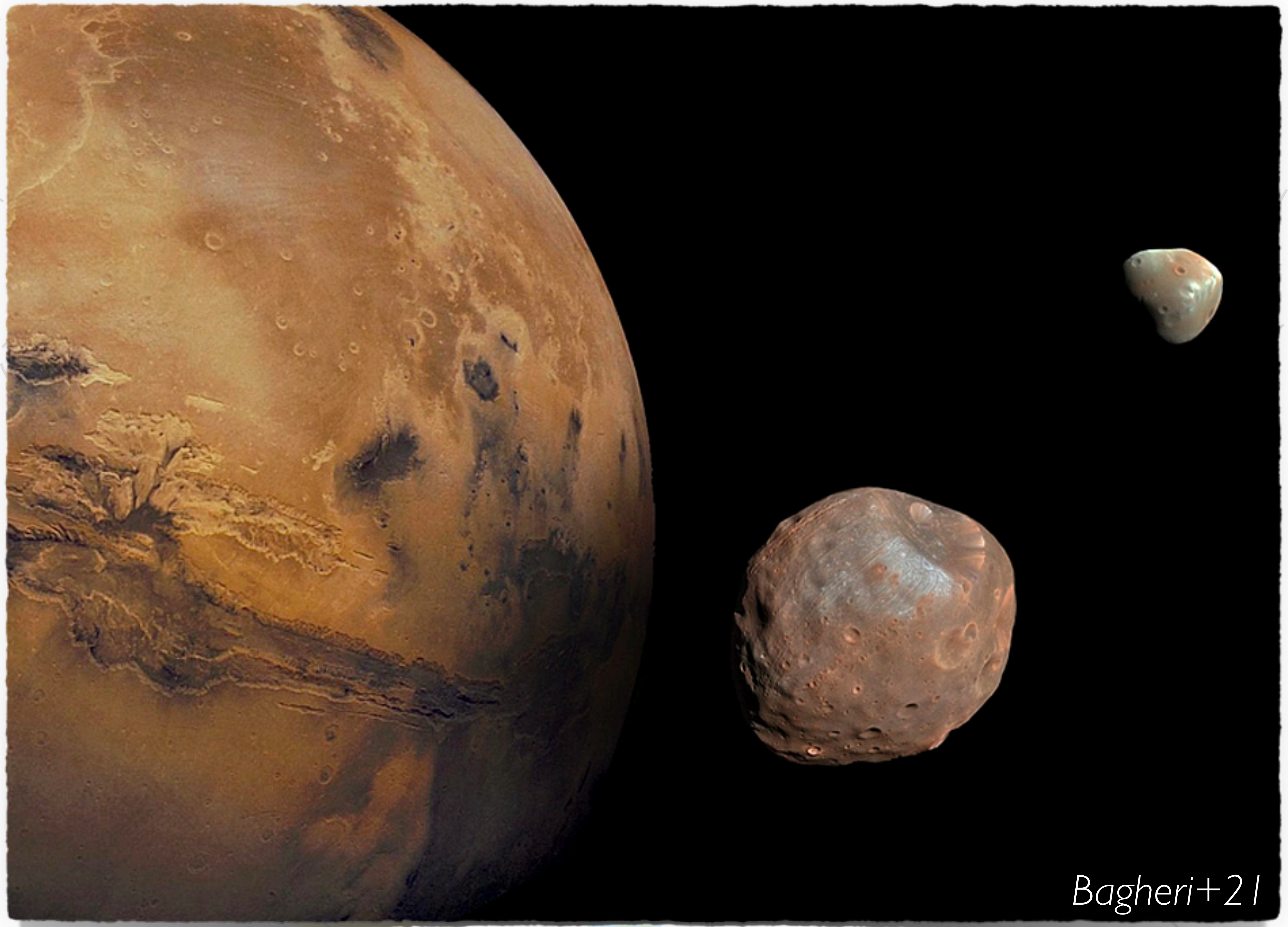
Ex: Mercury has $P_{rot} = 2/3\ P_{orb}$

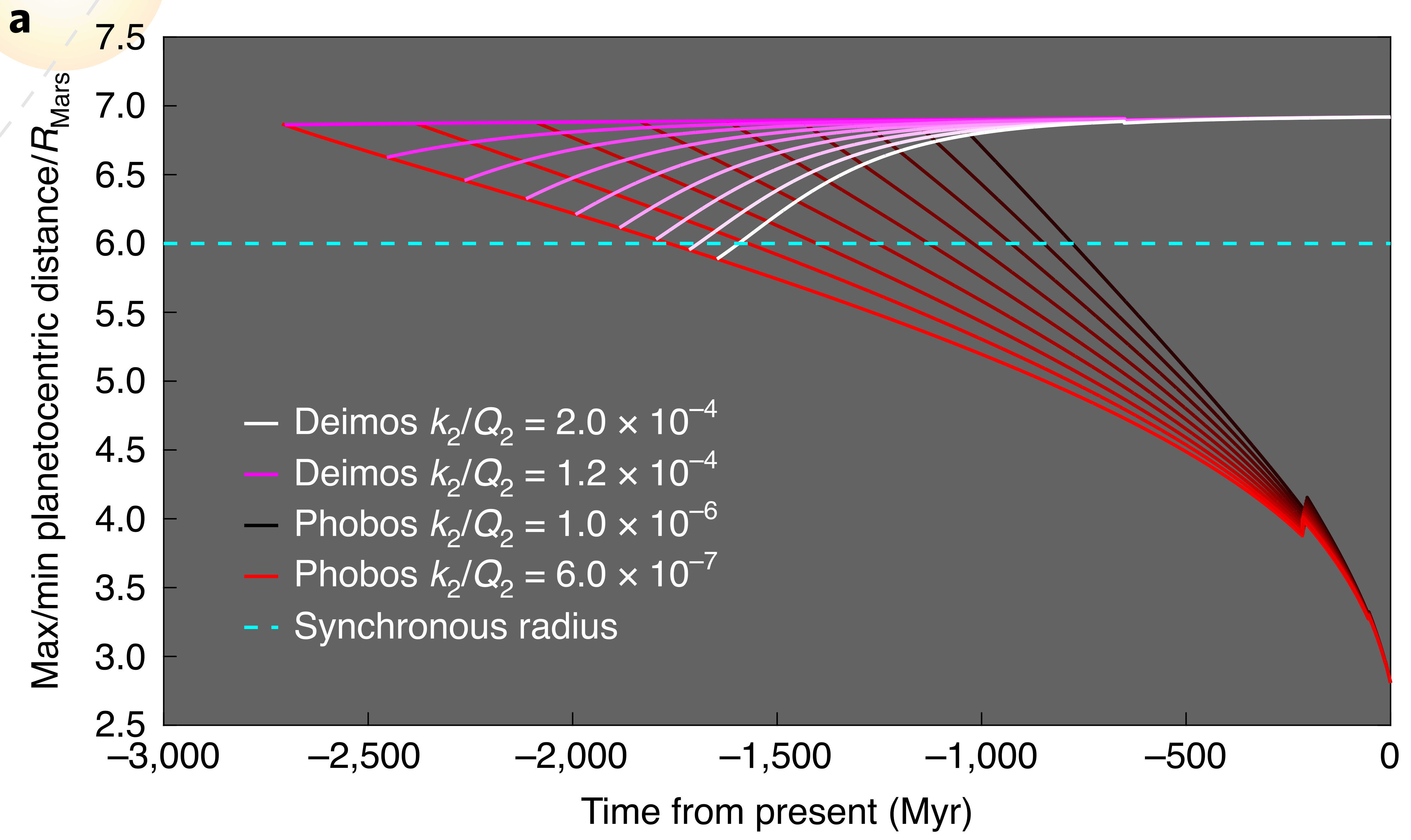


Planetary tide: rocky planets/cores

Tidal response → Orbital/rotational evolution





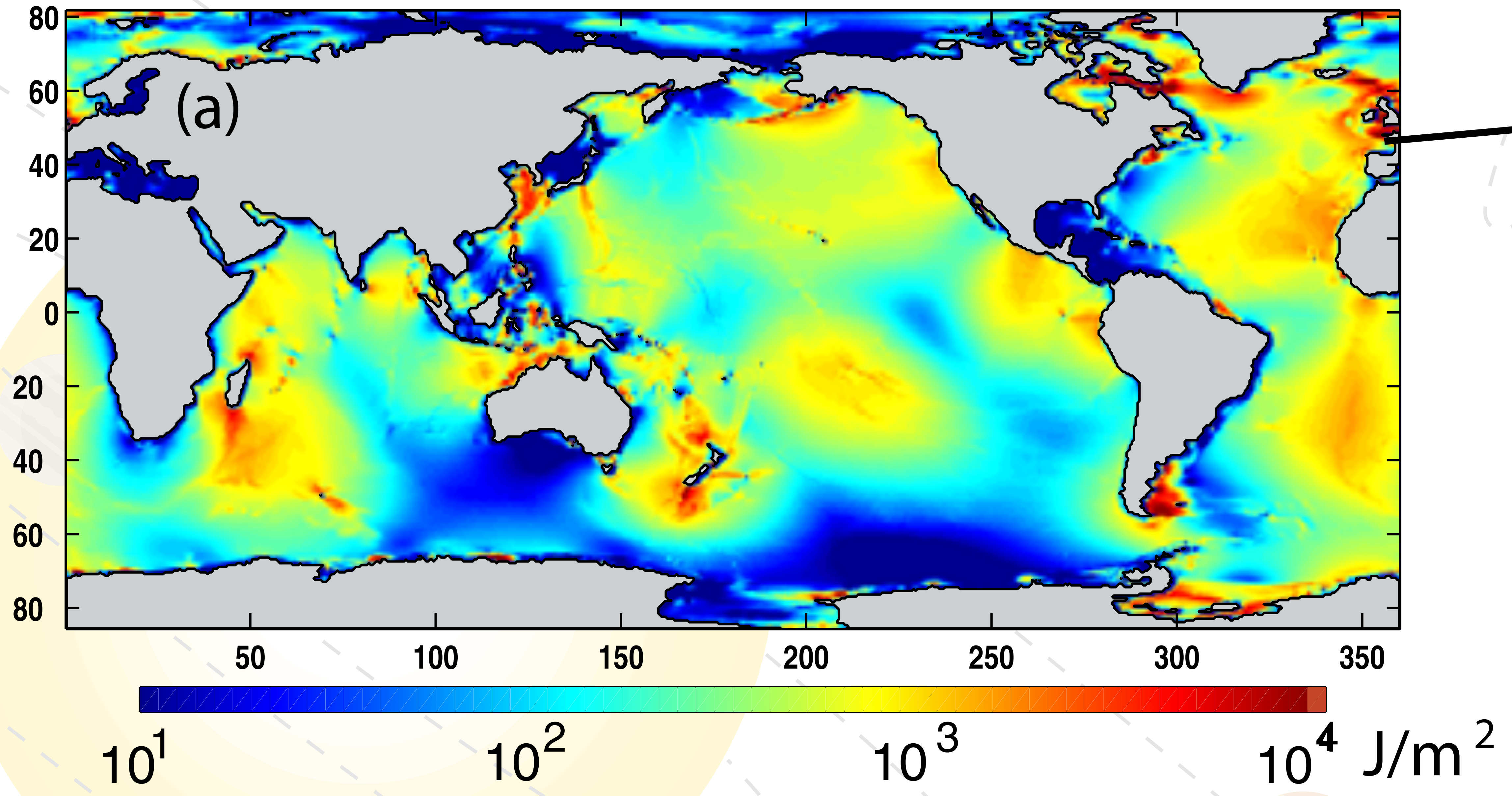


Dynamical evidence for **Phobos** and **Deimos** as **remnants** of a **disrupted common progenitor**

Planetary tide: rocky planets/liquid layers

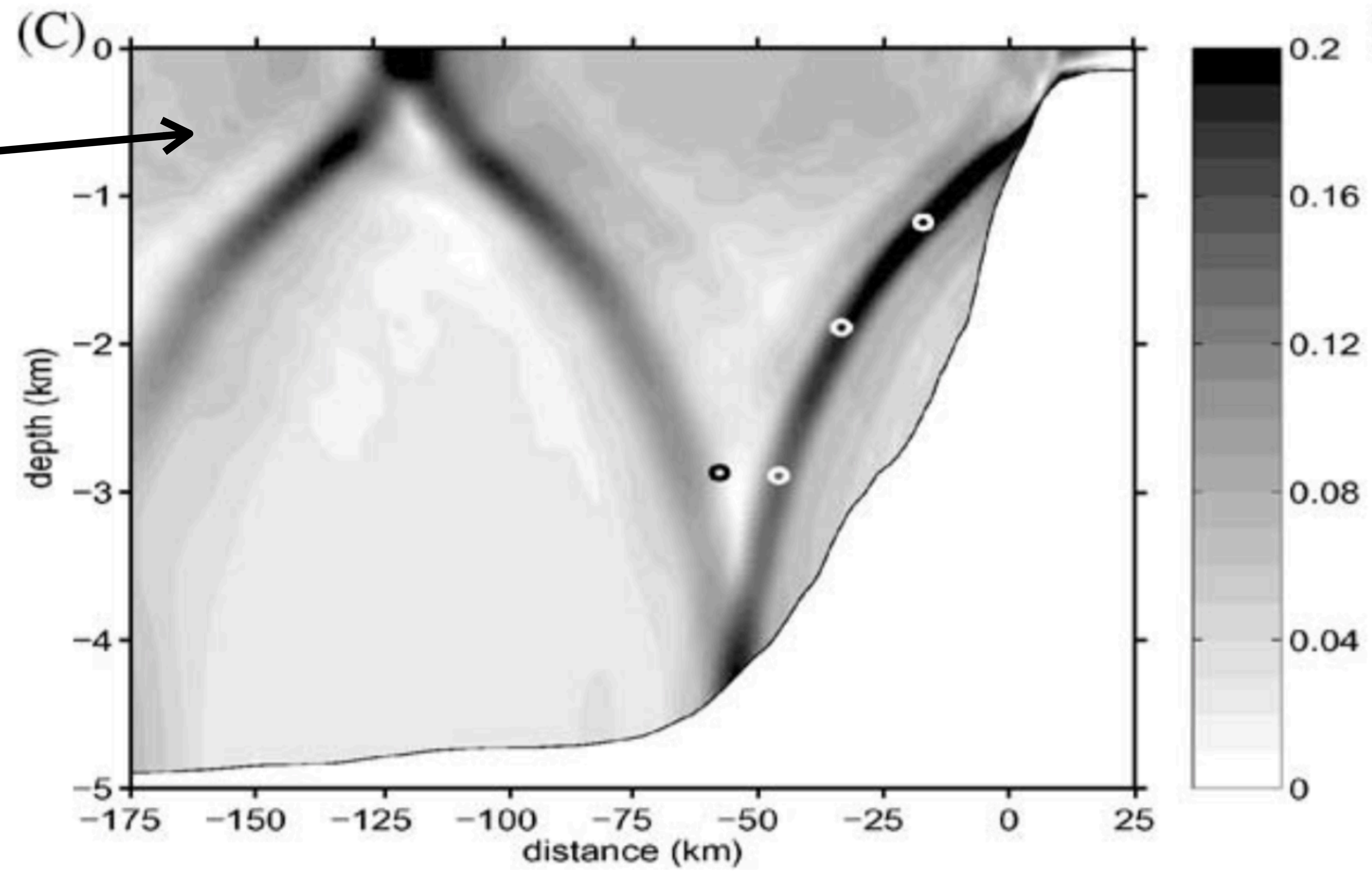
Rheology → Tidal response

Main **semi-diurnal component** from measurements from **TOPEX/Poseidon** altimeter data



[Egbert & ray 2003]

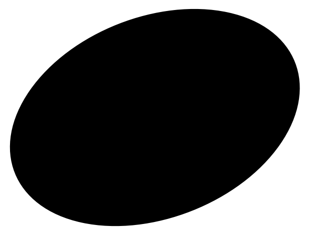
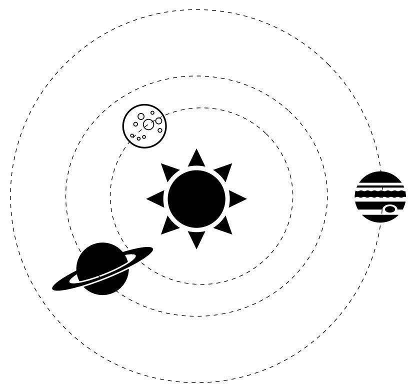
Internal gravity wave in the **bay of Biscay**



[Gerkema, Lam & Maas 2004]

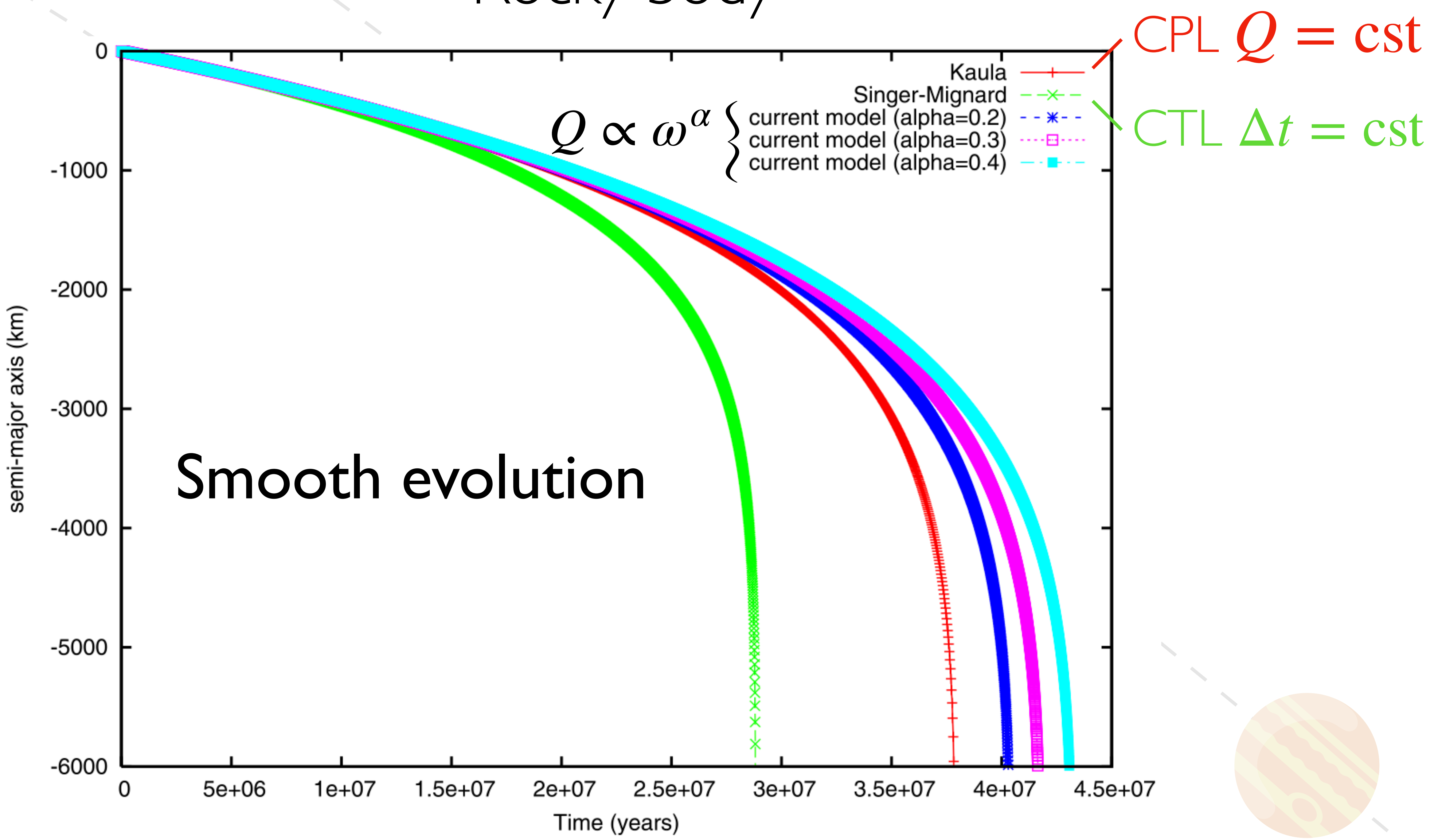
The **major component** of **tidal dissipation** for the Earth comes from the **ocean** (especially in shallow regions). **Without oceans** the **overall dissipation** of the Earth would be **1/10th of what it is today.**

Planetary tide: rocky planets/liquid layers

Tidal response  → Orbital/rotational evolution 

Evolution of Phobos's semi-major axis

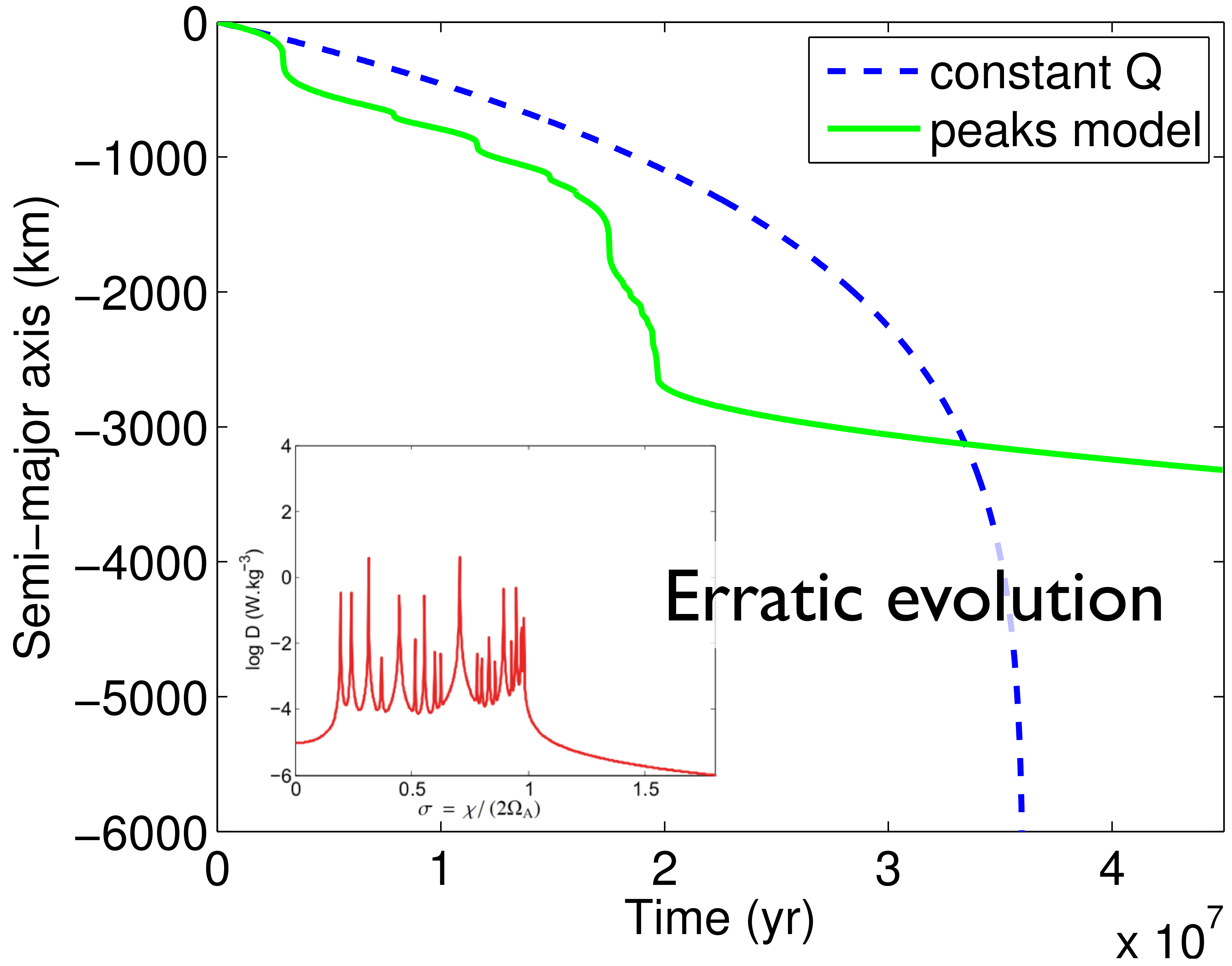
Rocky body



[Efroimsky & Lainey 2007]

Evolution of Phobos's semi-major axis

Fluid body

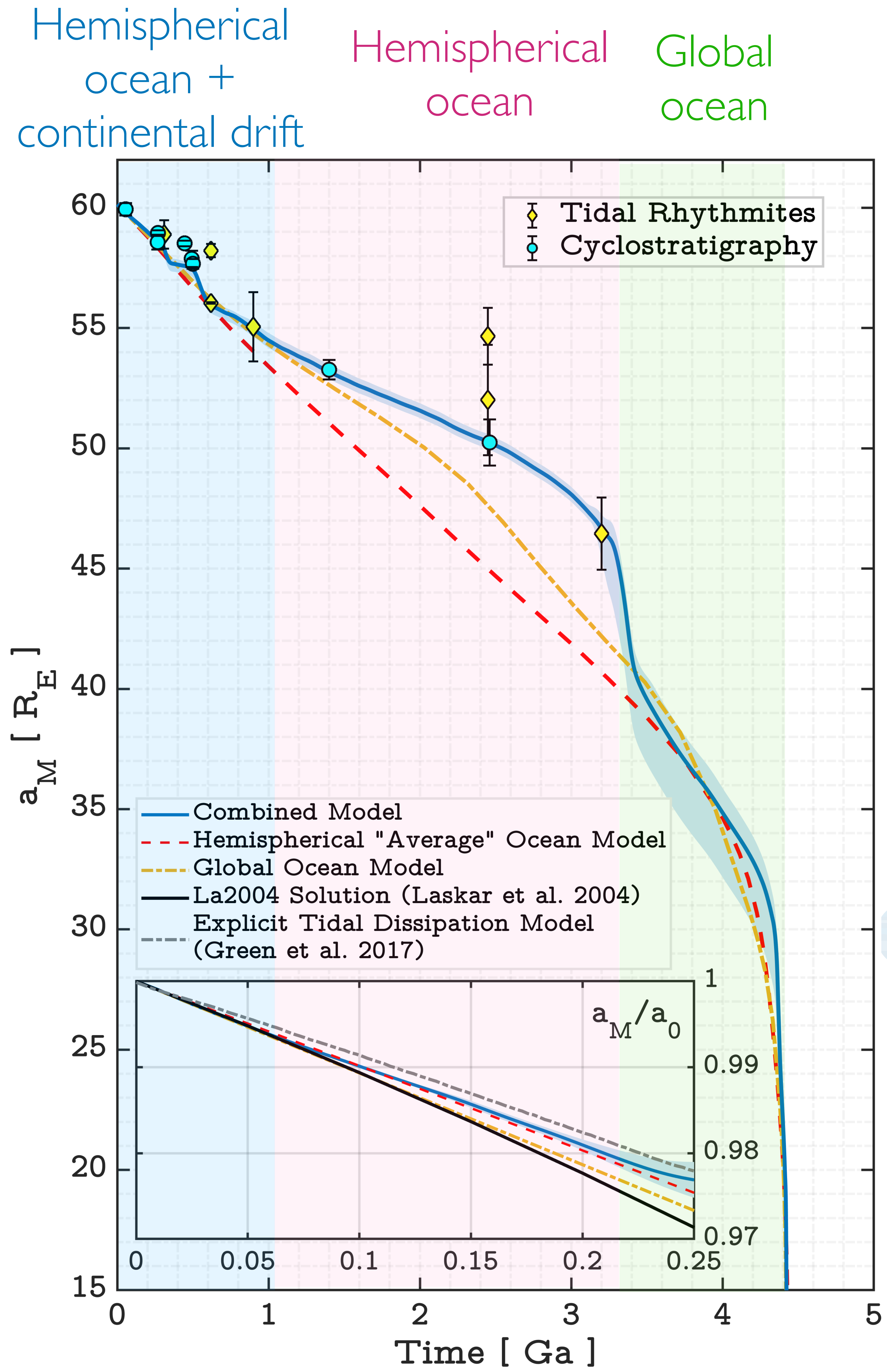


[Auclair-Desrotour+14]

Planetary tide: rocky planets/liquid layers

Earth-Moon system

[Farhat+22]



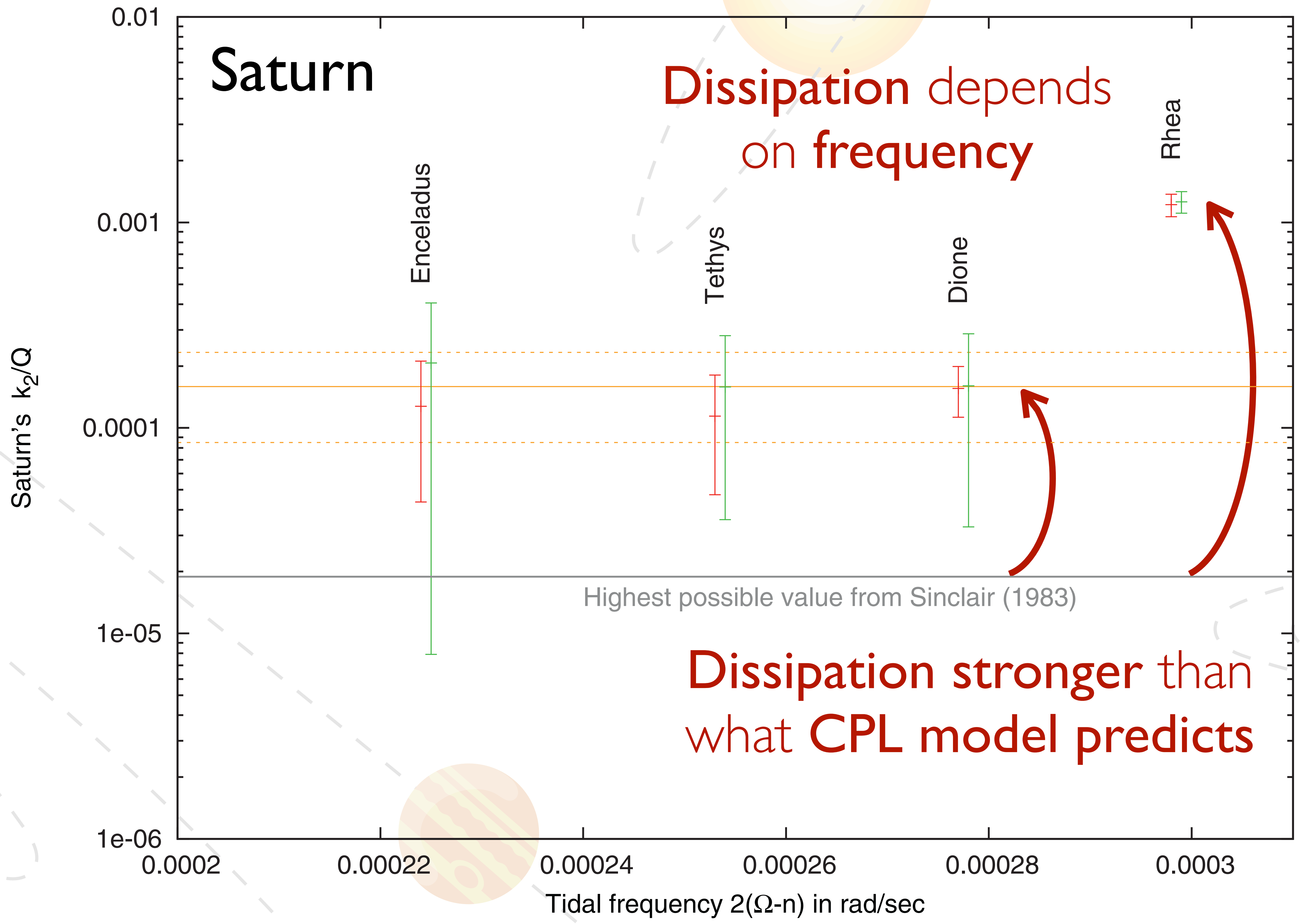
Complex evolution with multiple **crossings of resonances**

Reproduces well the data!

Planetary tide: gas giants

Tidal response ⟵ Orbital/rotational evolution

Constraints from the Solar System

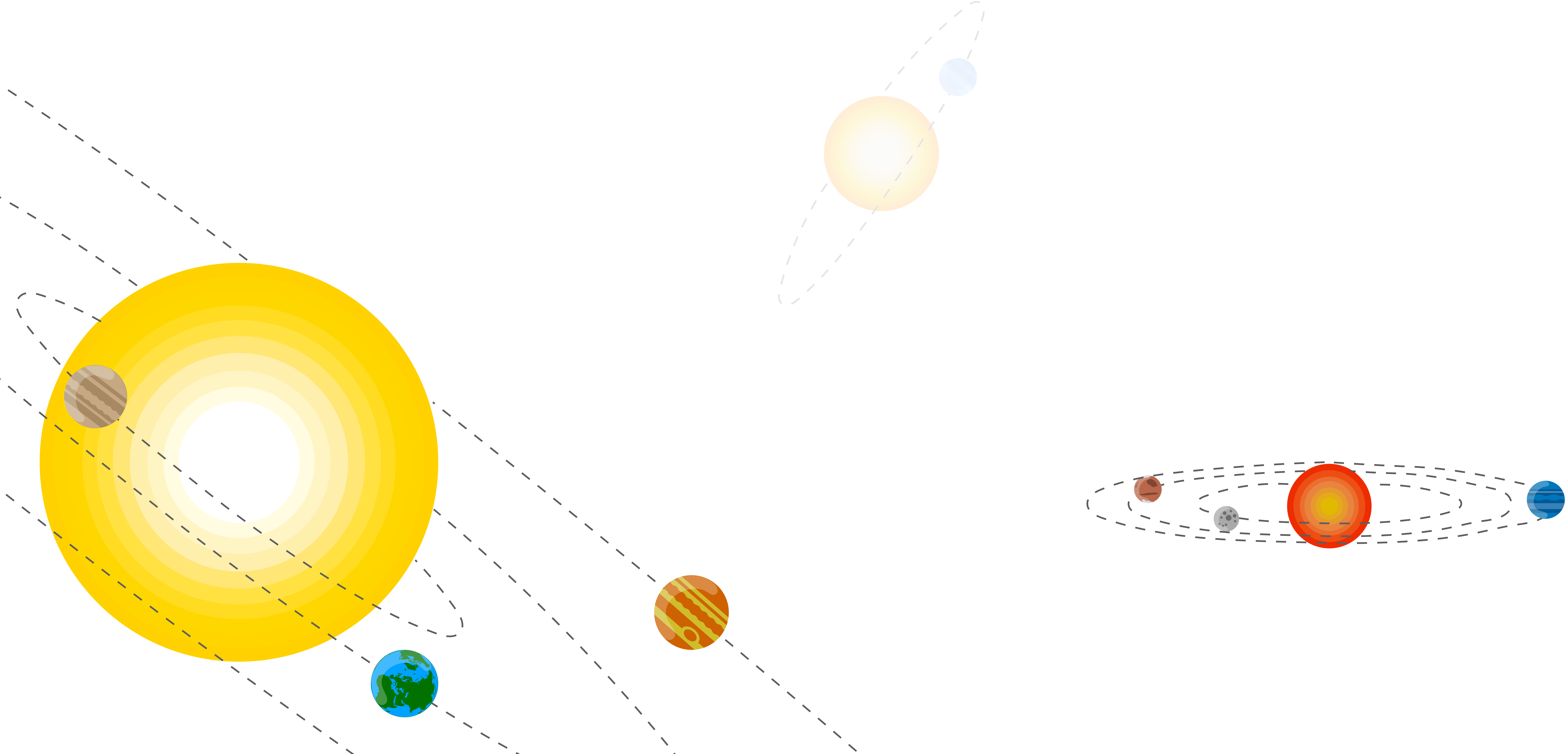


[Lainey+09, 12, ***17****]*

Tidal interactions

- Why are tides important?
- A bit of theory
  - ★ Tides, the easy way
  - ★ Tides, the hard way…
- Some constraints brought by studying tides…
  - ★ Stars
  - ★ Planets
  - ★ Multi-planet systems

Tides in multiplanet systems



Tides in N-body systems

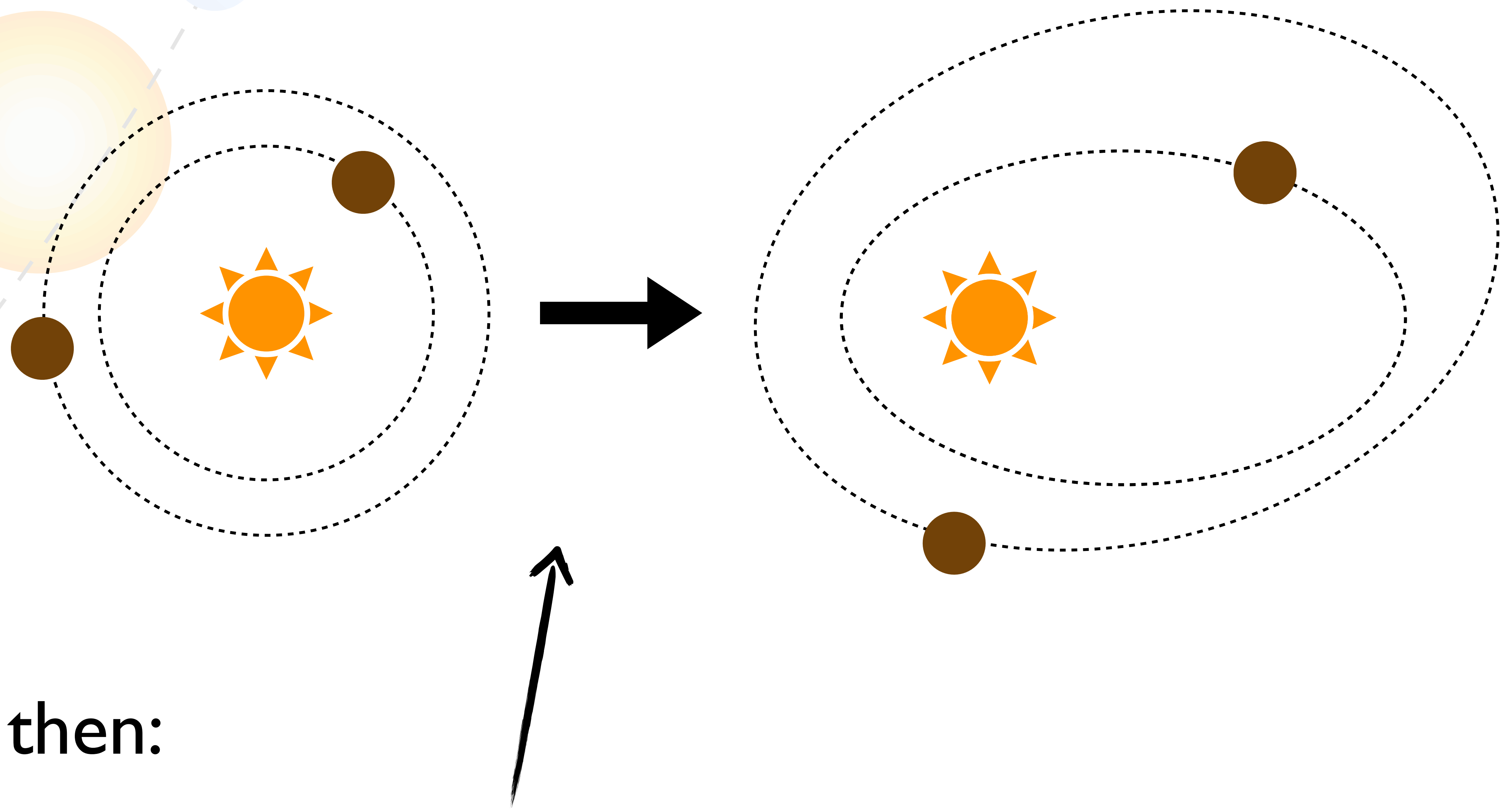
Tidal evolution for **multiple planet-systems**:

If equilibrium is possible *[*$L_{orb} > 3/4\ (L_{orb}+L_{rot})$, *Hut, 1980]*, **then:**

- Eccentricity = 0
- Planetary spin and orbital angular momentum aligned
- Planetary spin synchronized

Tides in N-body systems

Tidal evolution for **multiple planet-systems**:



If equilibrium is possible *[*$L_{orb} > 3/4\ (L_{orb}+L_{rot})$, *Hut, 1980]*, **then:**

- Eccentricity = 0
- Planetary spin and orbital angular momentum aligned
- Planetary spin synchronized

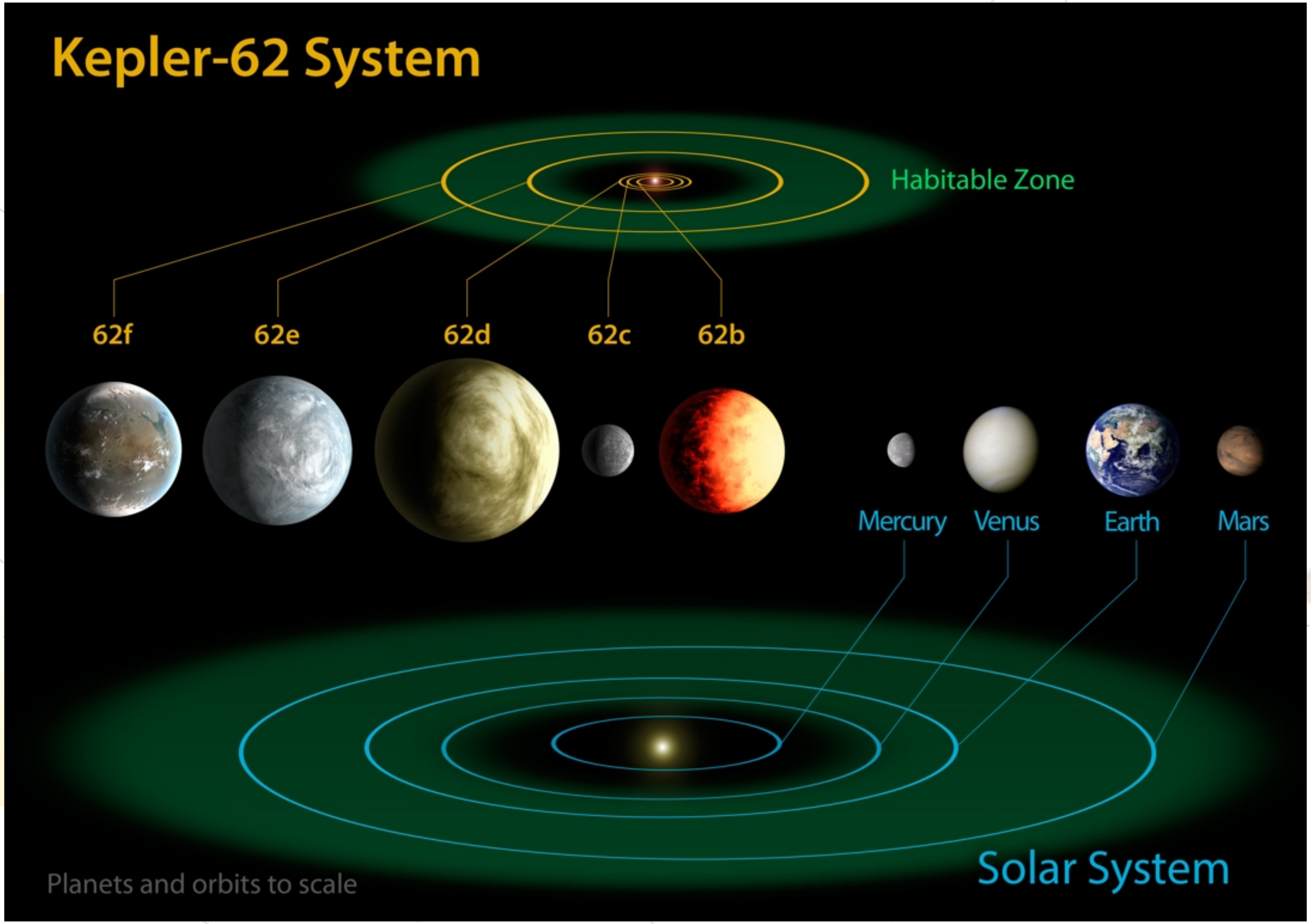
➜ Eccentricity reaches an **equilibrium** between tidal **damping** and planet-planet **excitation**

➜ Obliquity reaches an **equilibrium**

➜ Rotation depends on **eccentricity** and is **influenced** by planet-planet **excitation**

Tides and stability

The assessment of **stability** depends on whether **tides** are taken into account



Star:
$M_\star = 0.69\ M_\odot$

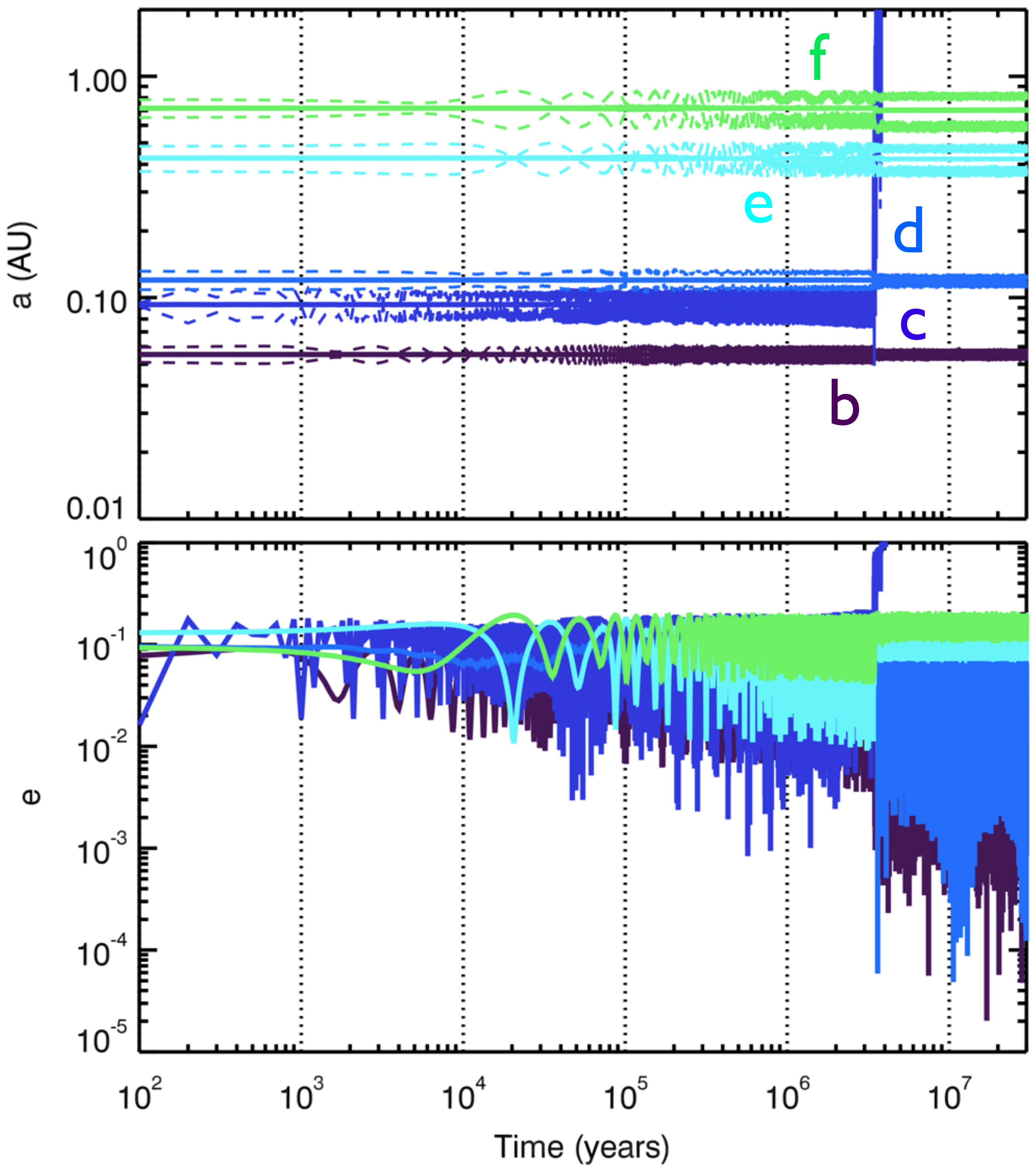
5 planets:
$0.54 < R_p/R_\oplus < 1.95$
$0.05 < a/AU < 0.72$

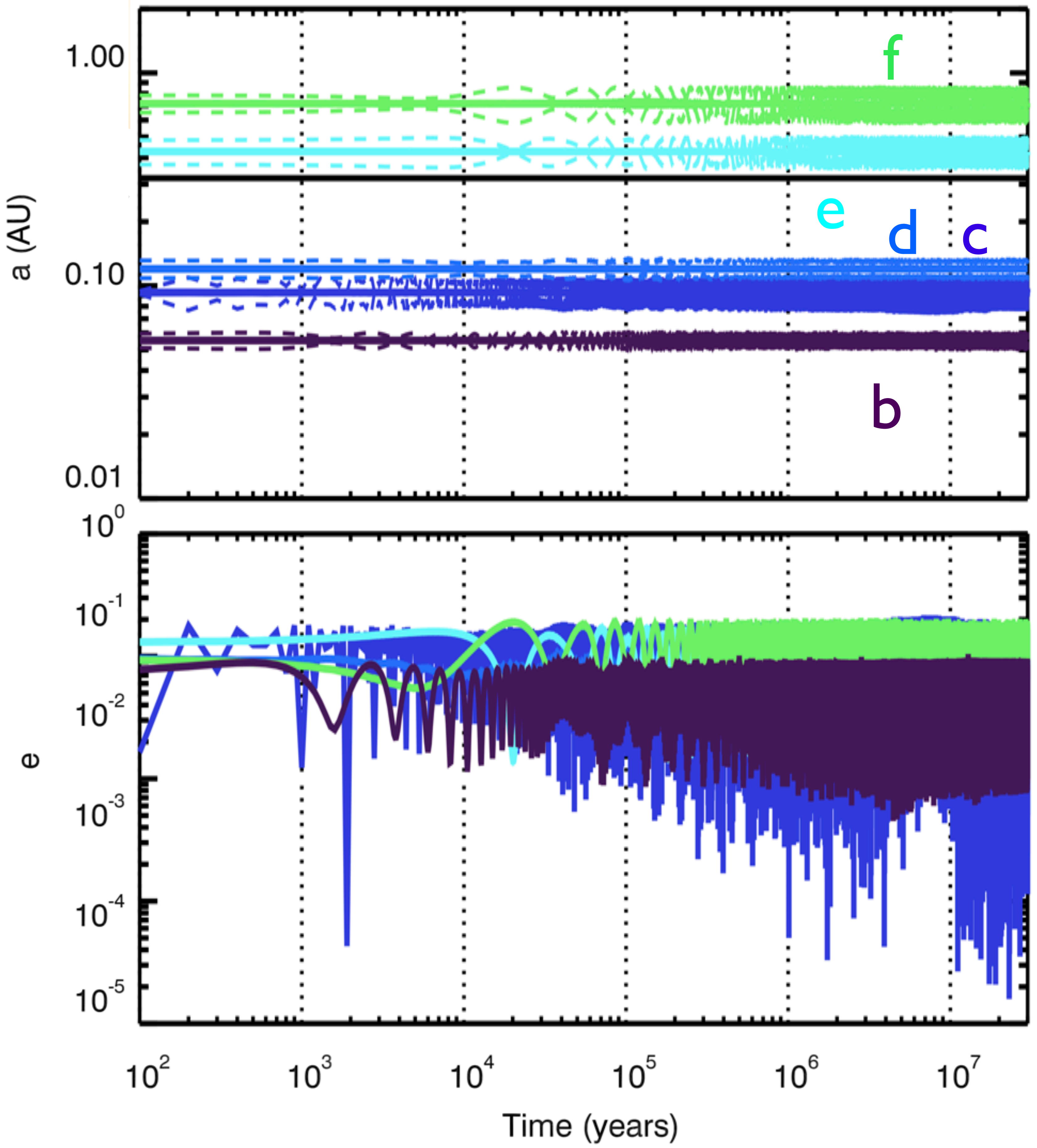
Borucki+13

Tides and stability

The assessment of **stability** depends on whether **tides** are taken into account

Pure N-body simulation *vs* **Simulation** with **Tides** and **GR**





Bolmont+15

Constraining the eccentricity

Star:

$M_\star$ = 0.95 $M_\odot$

5 planets:

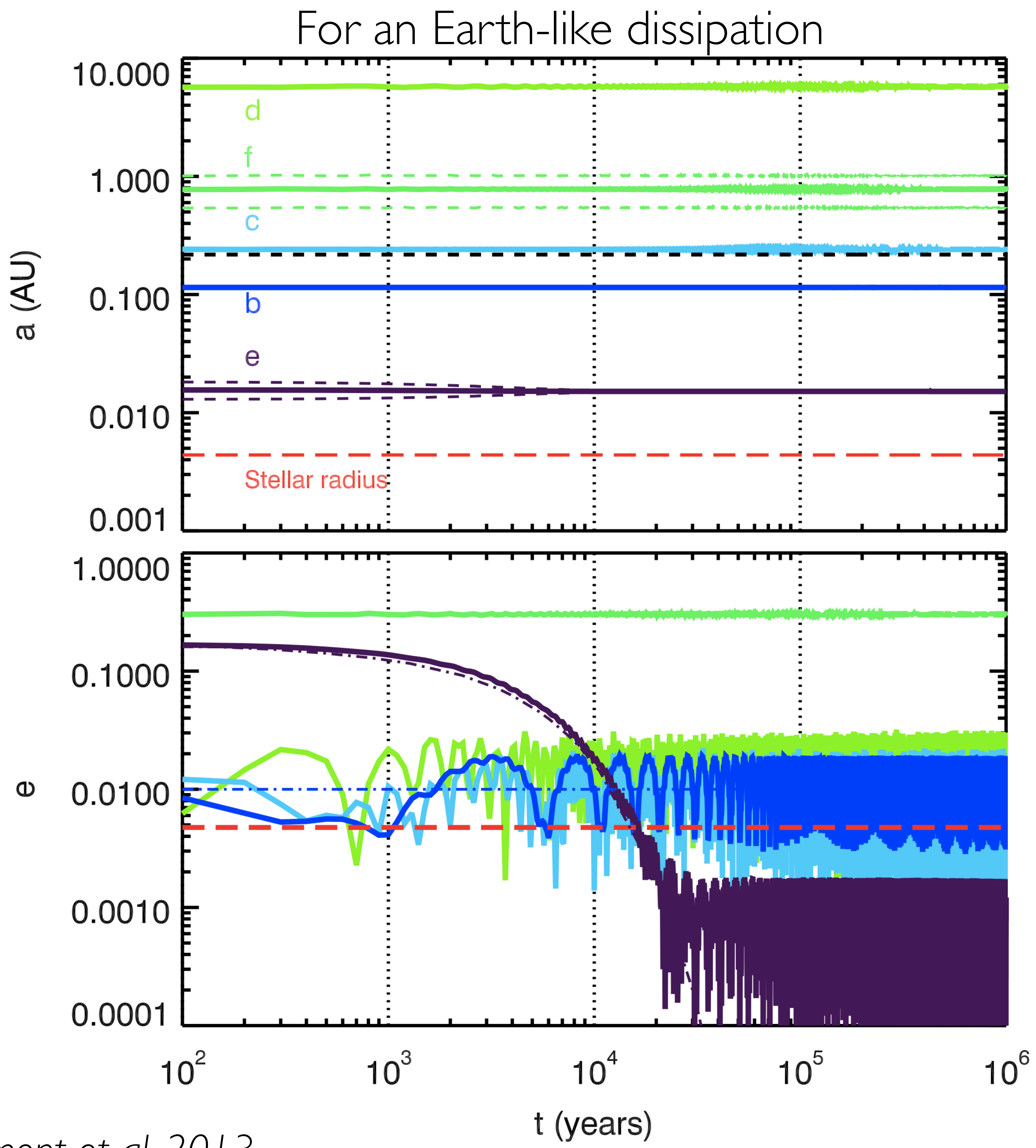
0.015 < a/AU < 5.4

Demory+11

55 Cancri

http://www.spacetelescope.org/images/heic1603a/

Constraining the eccentricity



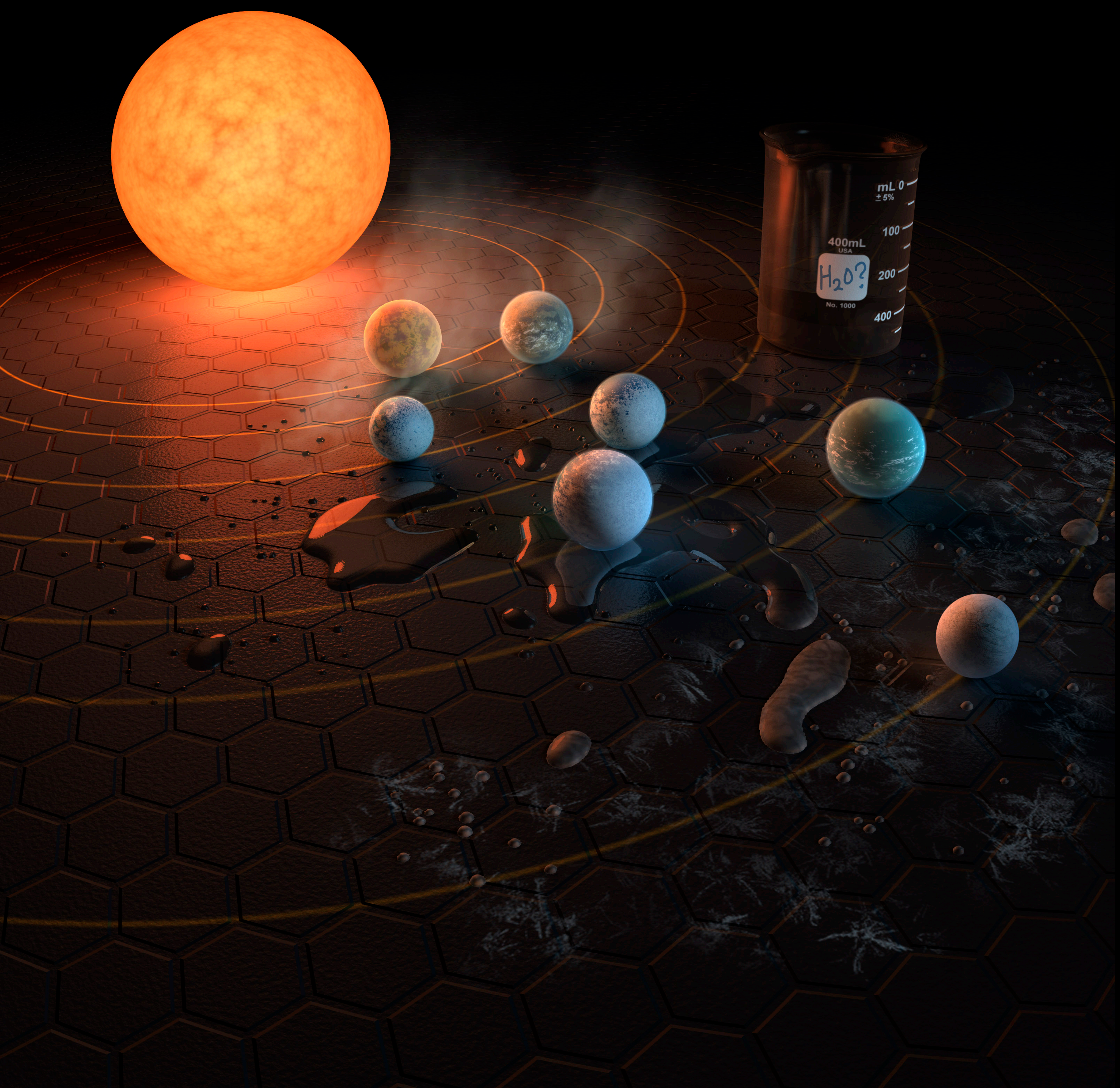
Orbital parameters from *Dawson & Fabrycky (2010)*

Eccentricity of **planet e** should be **lower** than **0.002**

→ **Equilibrium** between **planet-planet excitation** and **tidal damping**

Bolmont et al. 2013

Tides in N-body systems: TRAPPIST-1



Star:
$M_\star = 0.09\ M_\odot$

5 planets:
$0.76 < R_p/R_\oplus < 1.1$
$0.01 < a/\mathrm{AU} < 0.06$

Gillon+16,17

Constraining the eccentricity

All planets should have **eccentricities** lower than 0.01
Planets b & c are likely to have **eccentricities** lower than 0.001

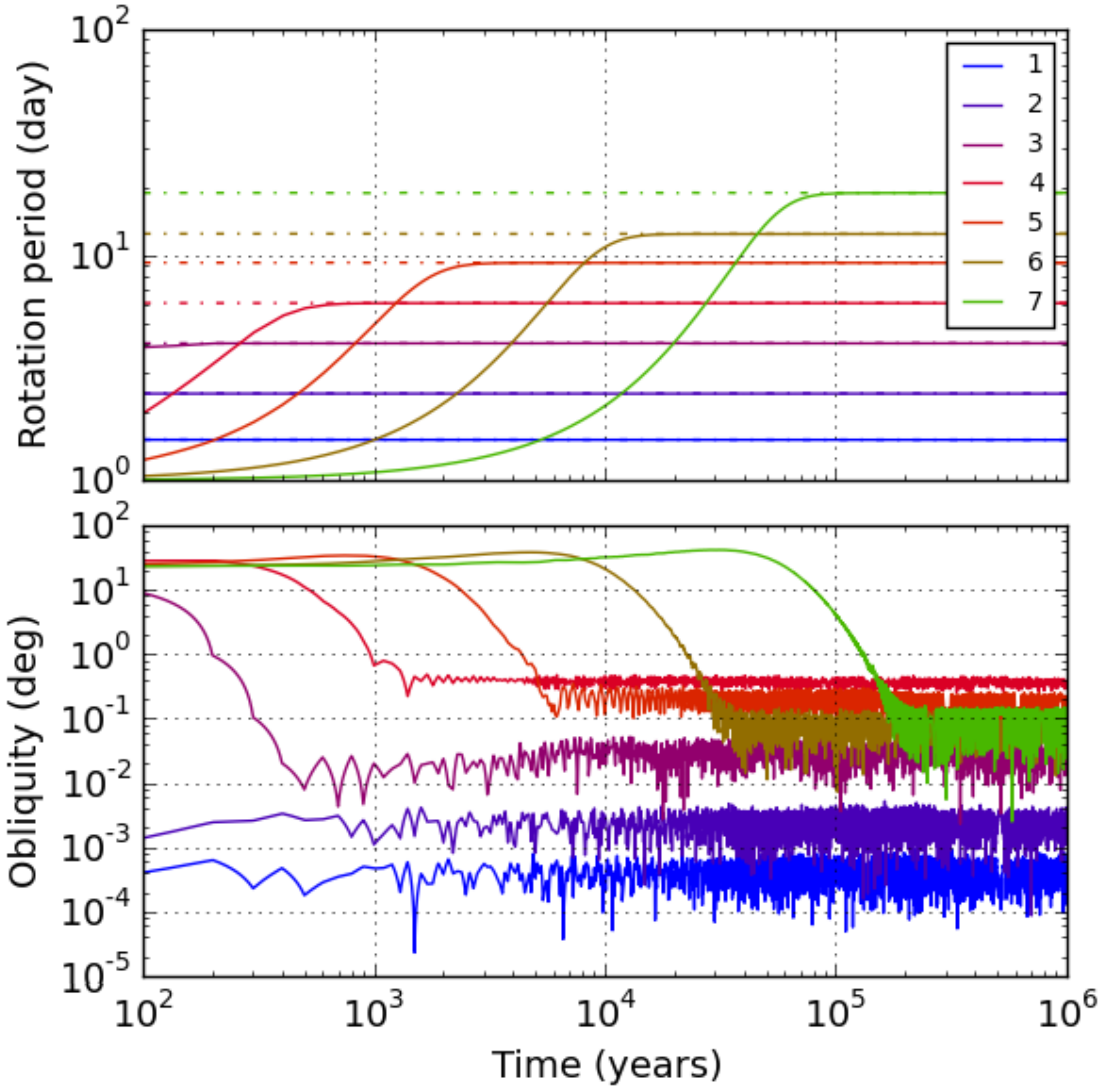
Turbet+18

Constraining the rotation

The rotation evolves very fast!

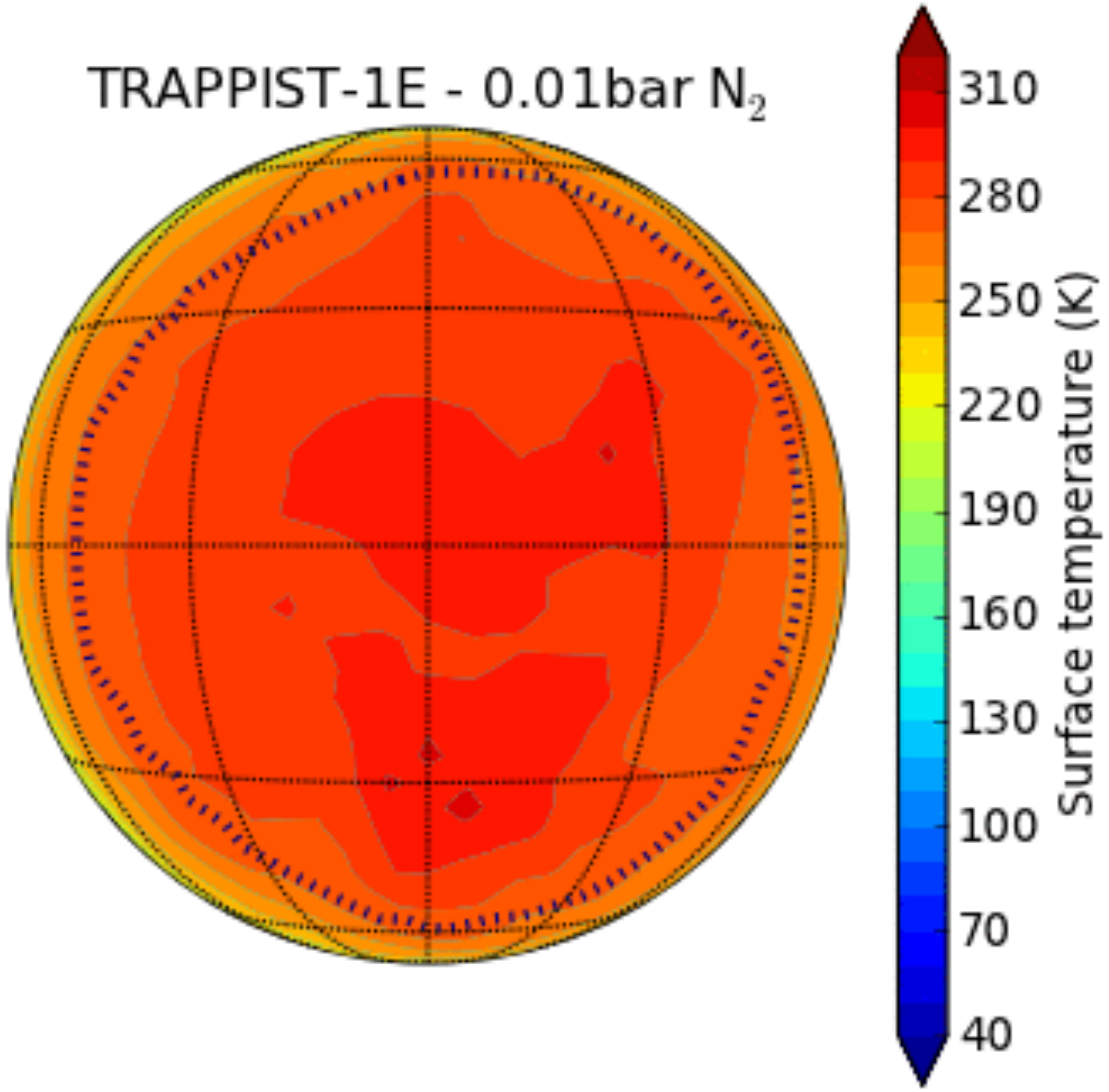
In ~200,000 yr:

- **Obliquity** damped
- **Rotation** pseudo-synchronized

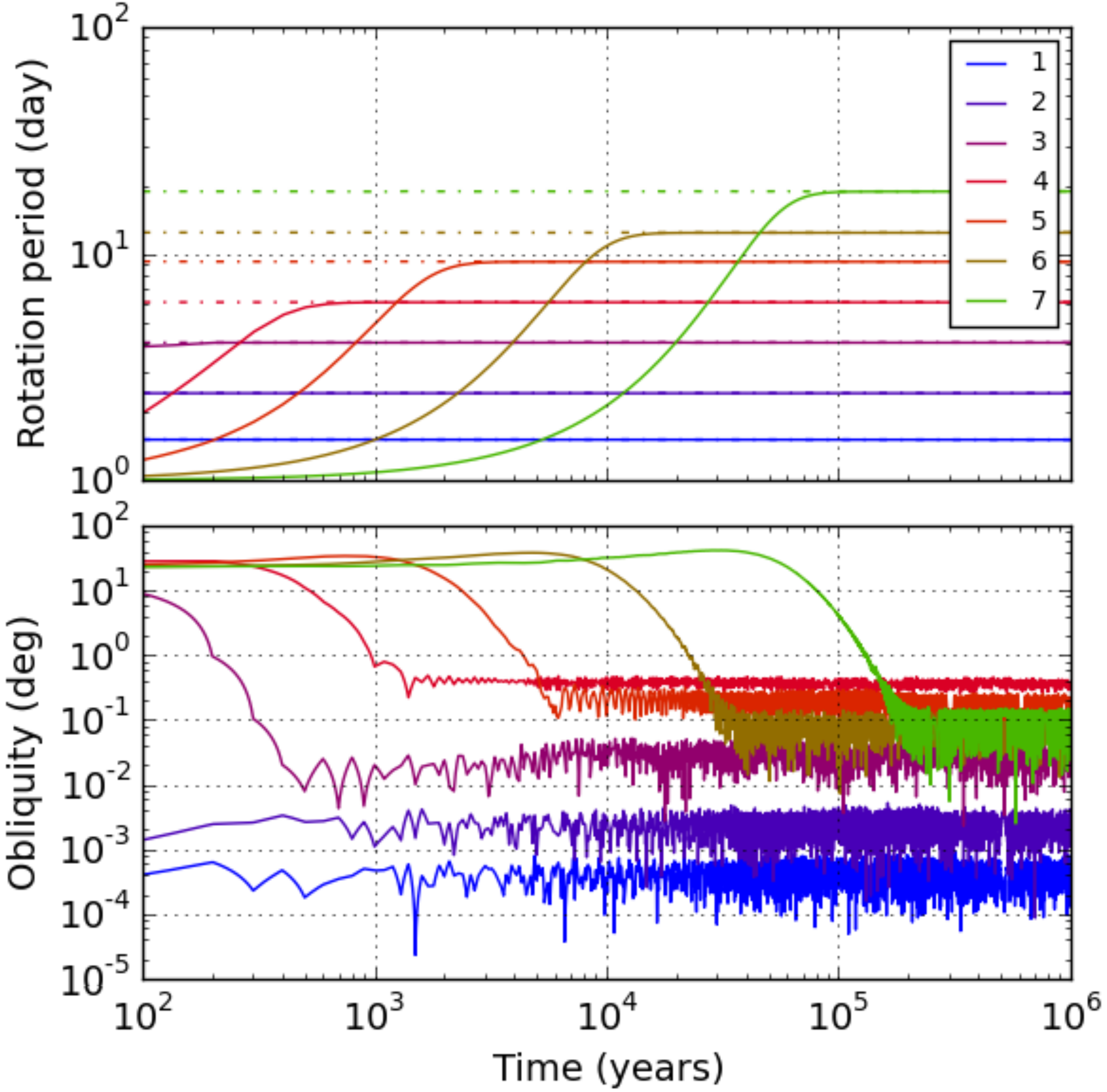


Constraining the rotation

➡ Impact on the **climate** of the **planets**



Turbet+18



The rotation evolves very fast!

In ~200,000 yr:

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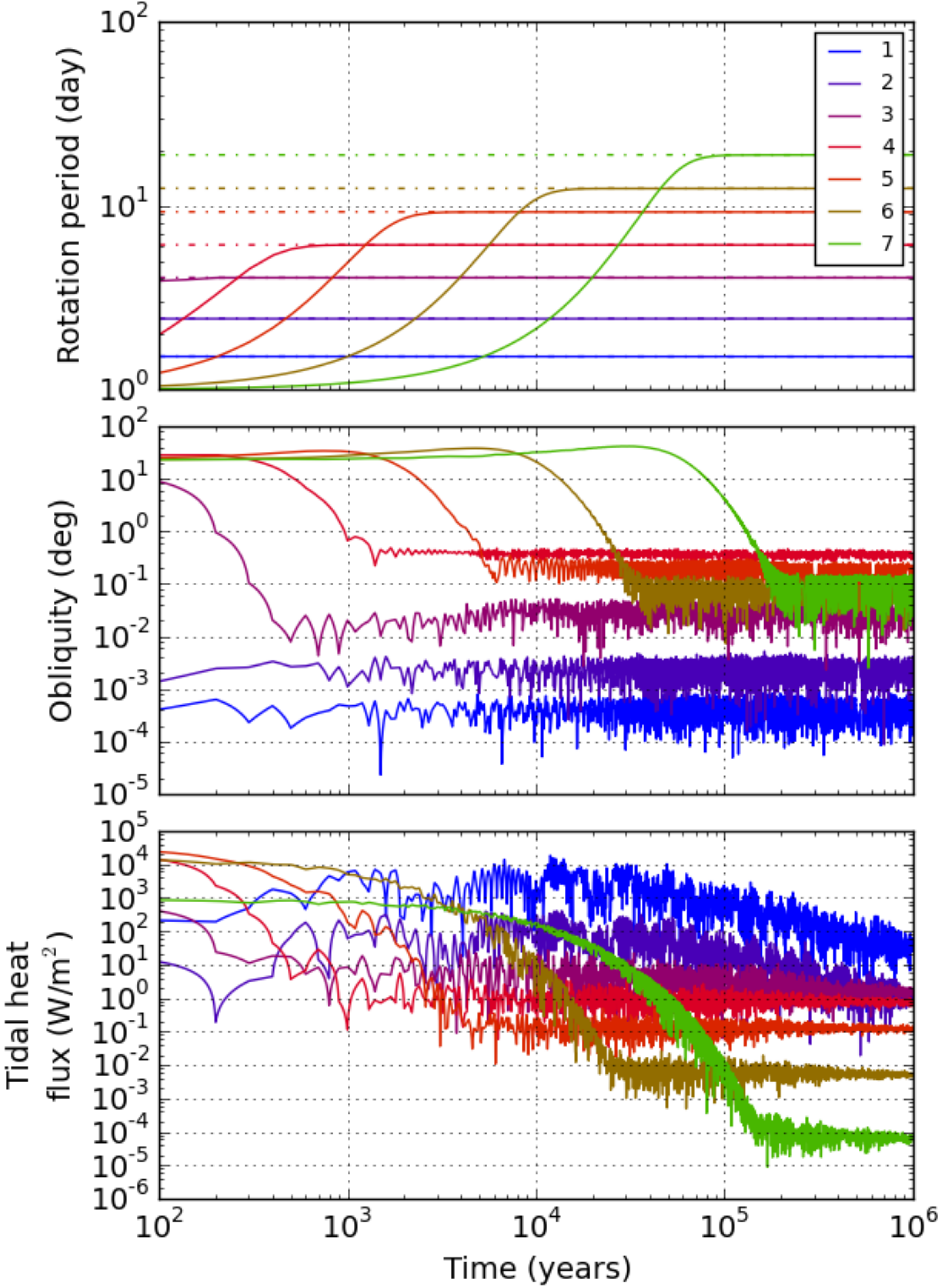
Constraining the tidal heat flux

Non zero eccentricity and obliquity
➡Tidal heat flux



Constraining the tidal heat flux

Tidal heating? Significant or not?

- Tidal **heat flux** using **CTL model** *[from N-body simulations with tides; Turbet+18]*

Parameter	Tb	Tc	Td	Te	Tf	Tg	Th	Unit
ecc mean ($\times10^{-3}$)	0.6	0.5	3.9	7.0	8.4	3.8	2.8	
ecc max ($\times10^{-3}$)	1.5	1.2	5.9	8.3	9.7	4.8	4.0	
Φ_{tid}mean	4.8	0.17	0.17	0.09	0.01	$< 10^{-3}$	$< 10^{-4}$	W m^{-2}
Φ_{tid}max	25	0.90	0.38	0.12	0.02	$< 10^{-3}$	$< 10^{-4}$	W m^{-2}

≈ Earth's heat flux *[Pollack+93]*

> Io's tidal heat flux *[Spencer+00]*

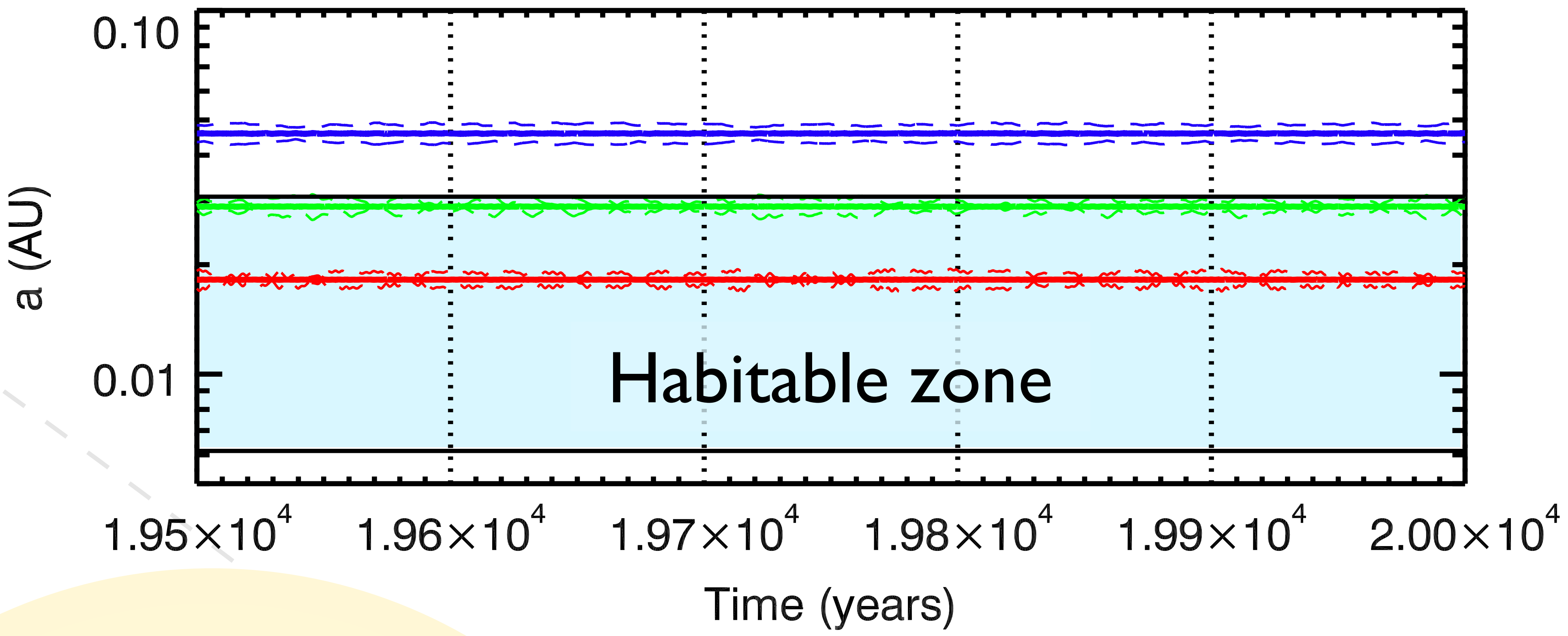
- Maximum tidal **heating** for uniform planets and **Maxwell rheology** *[Makarov+18]*

- Tidal **heat flux** using model for "uniform" planets and **Maxwell rheology** [uniform viscosity and rigidity based on each planet's composition; *Barr+18]*

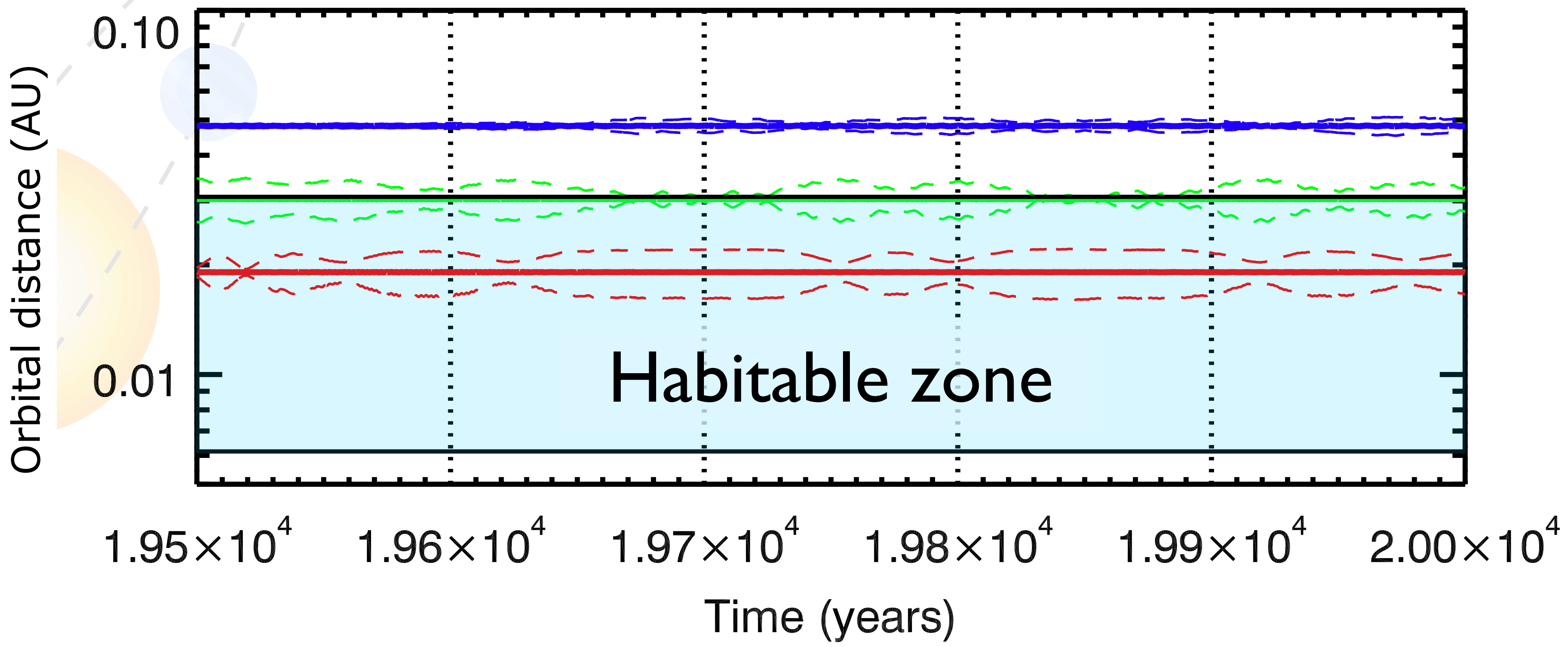
- Tidal **heat flux** using model with **multi-layer bodies** and **Andrade's rheology** *[Bolmont+20]*

Tidal heat flux and habitability

Non resonant system



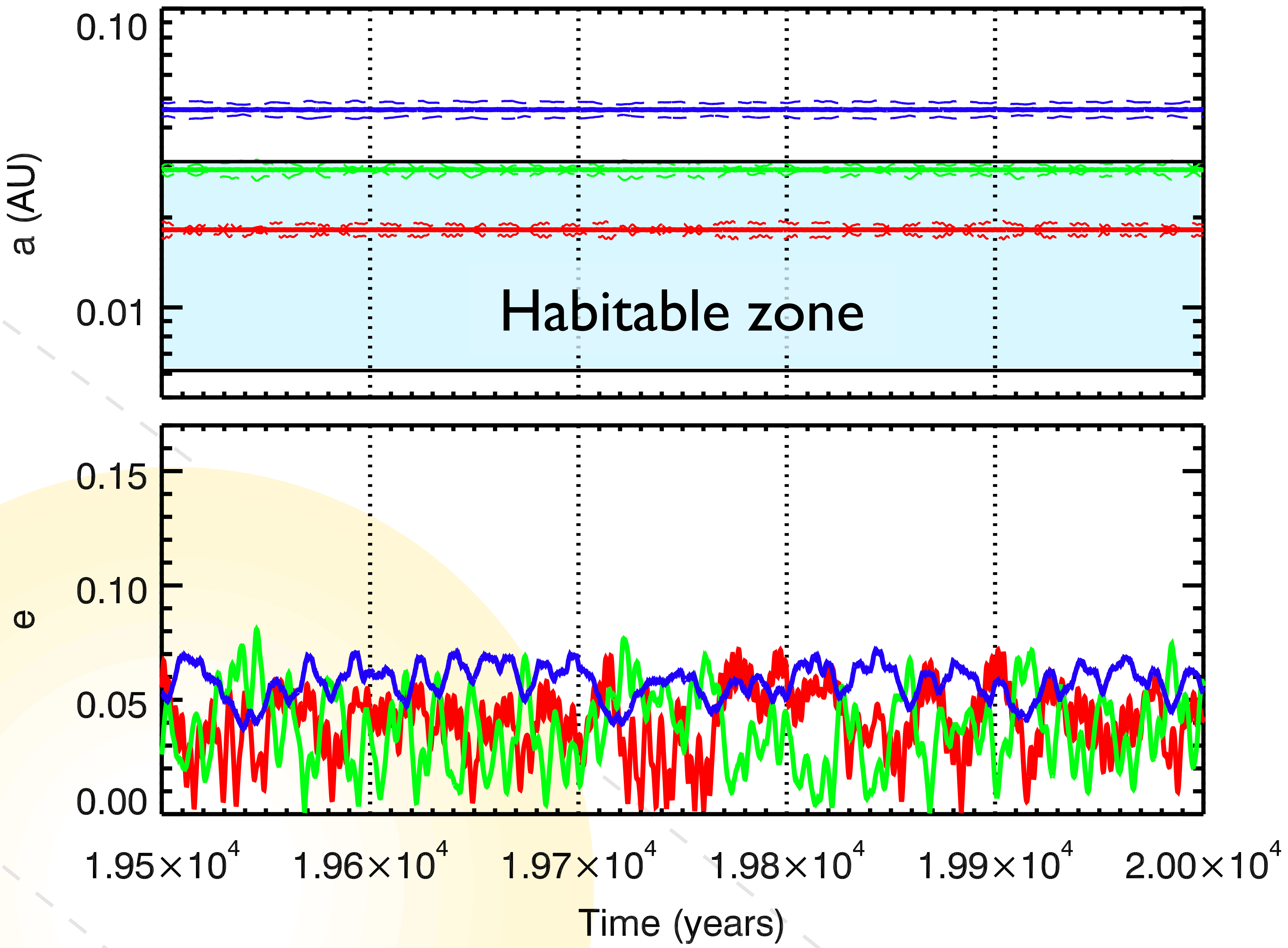
Resonance 2:1



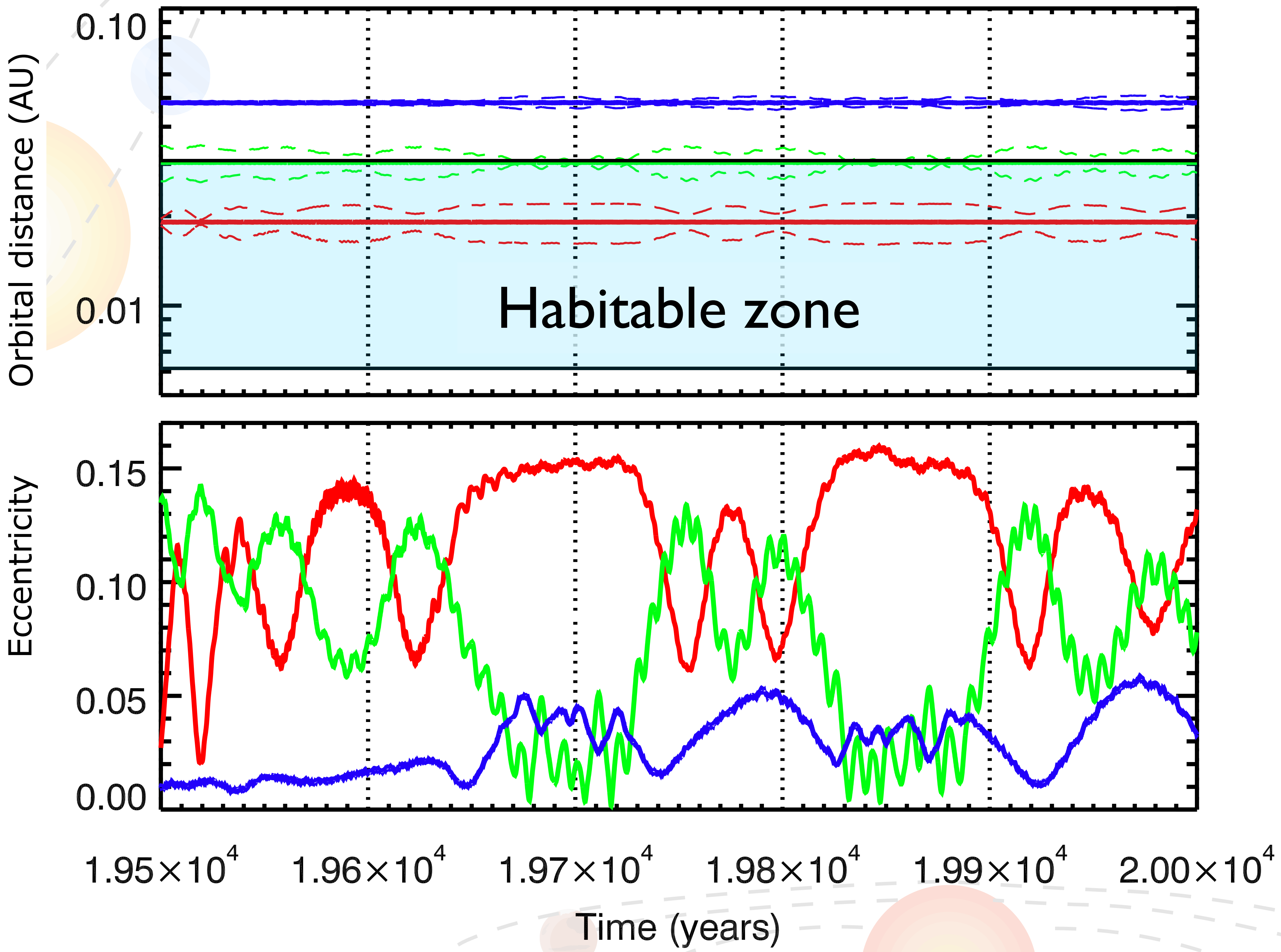
Brown dwarf and **3 Earth-like planets** *[Bolmont 2018]*

Tidal heat flux and habitability

Non resonant system



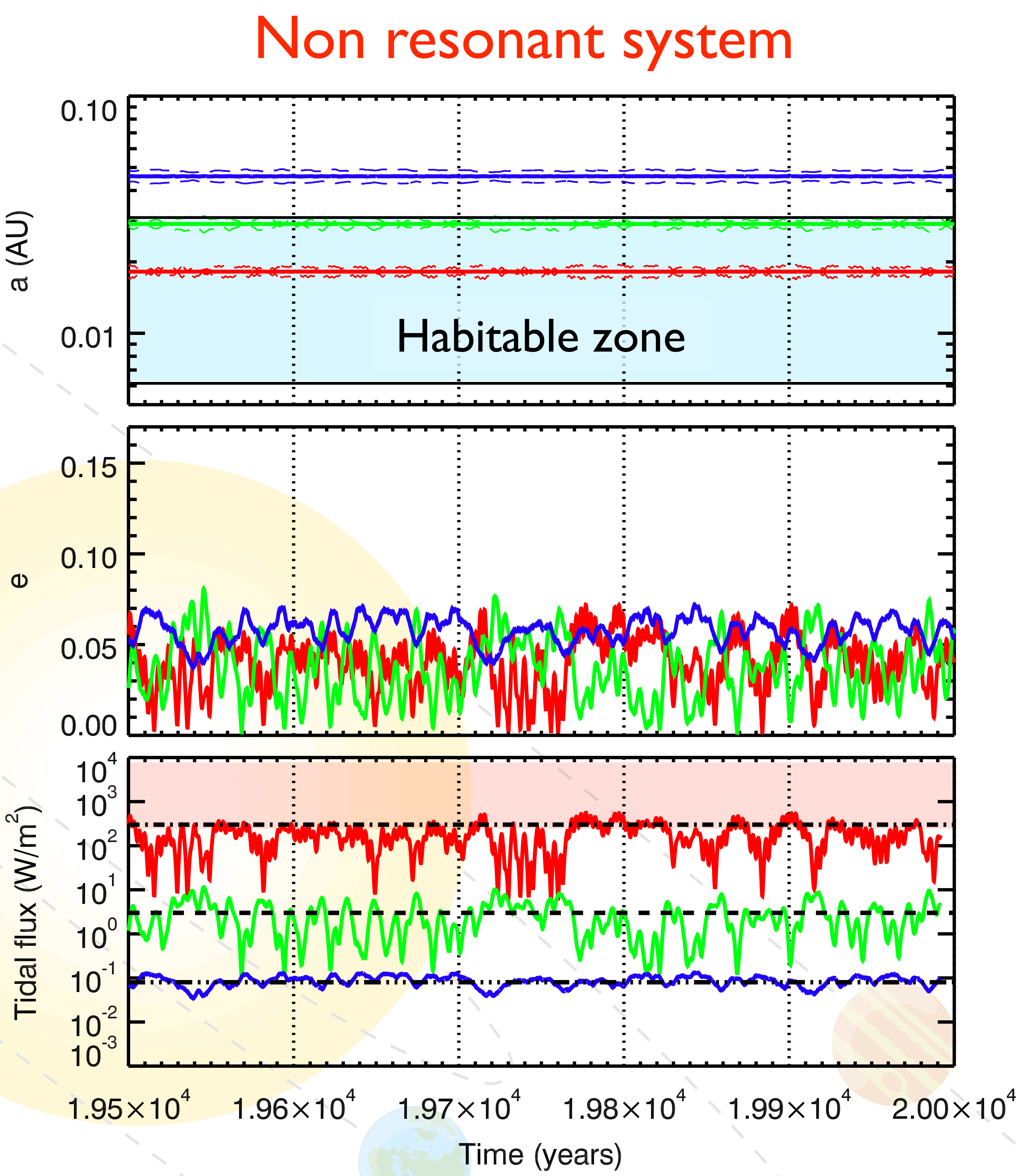
Resonance 2:1



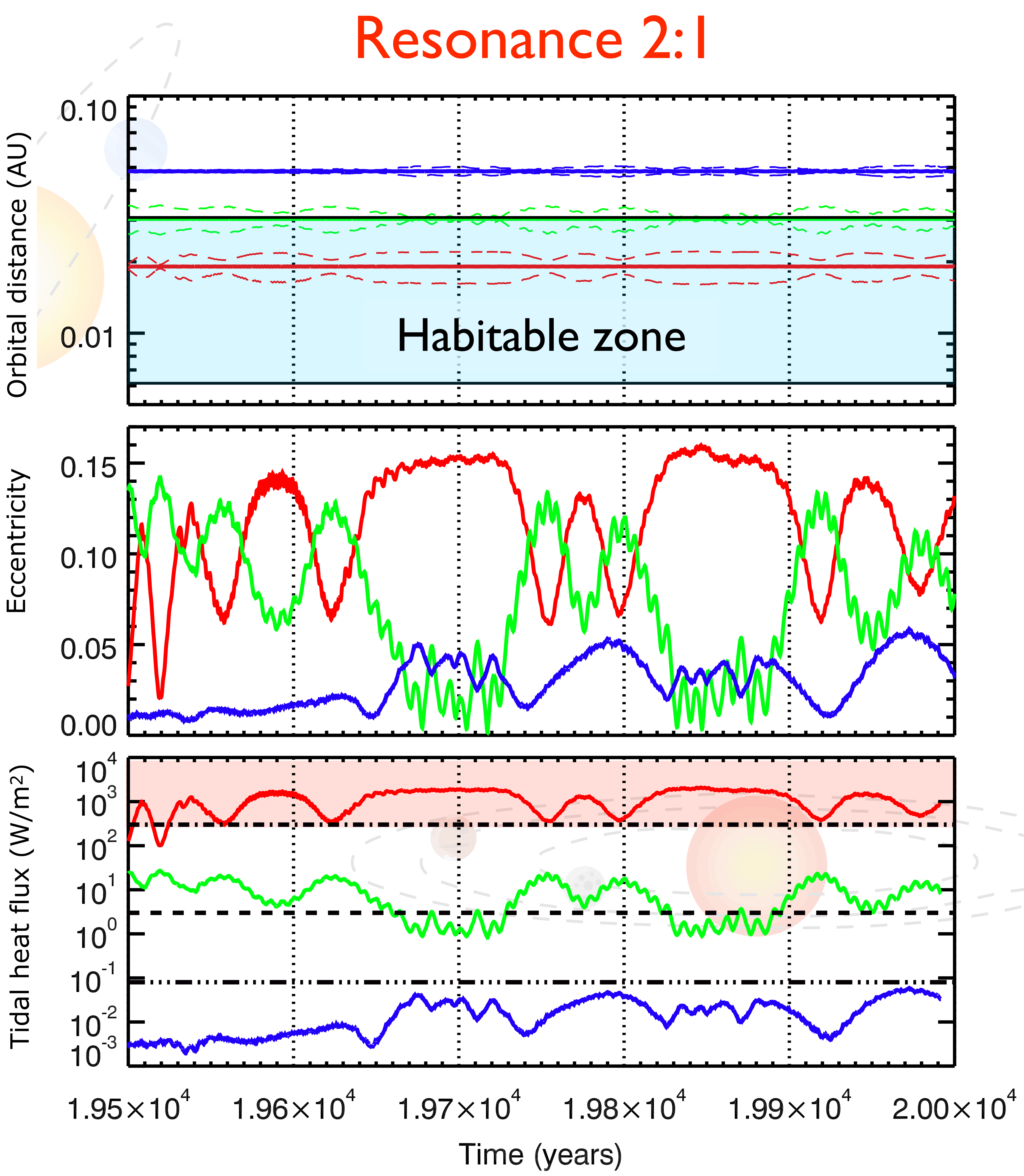
Bolmont 18

Also discussed in *Barnes+10* for planets around M-dwarfs

Tidal heat flux and habitability



Bolmont 18



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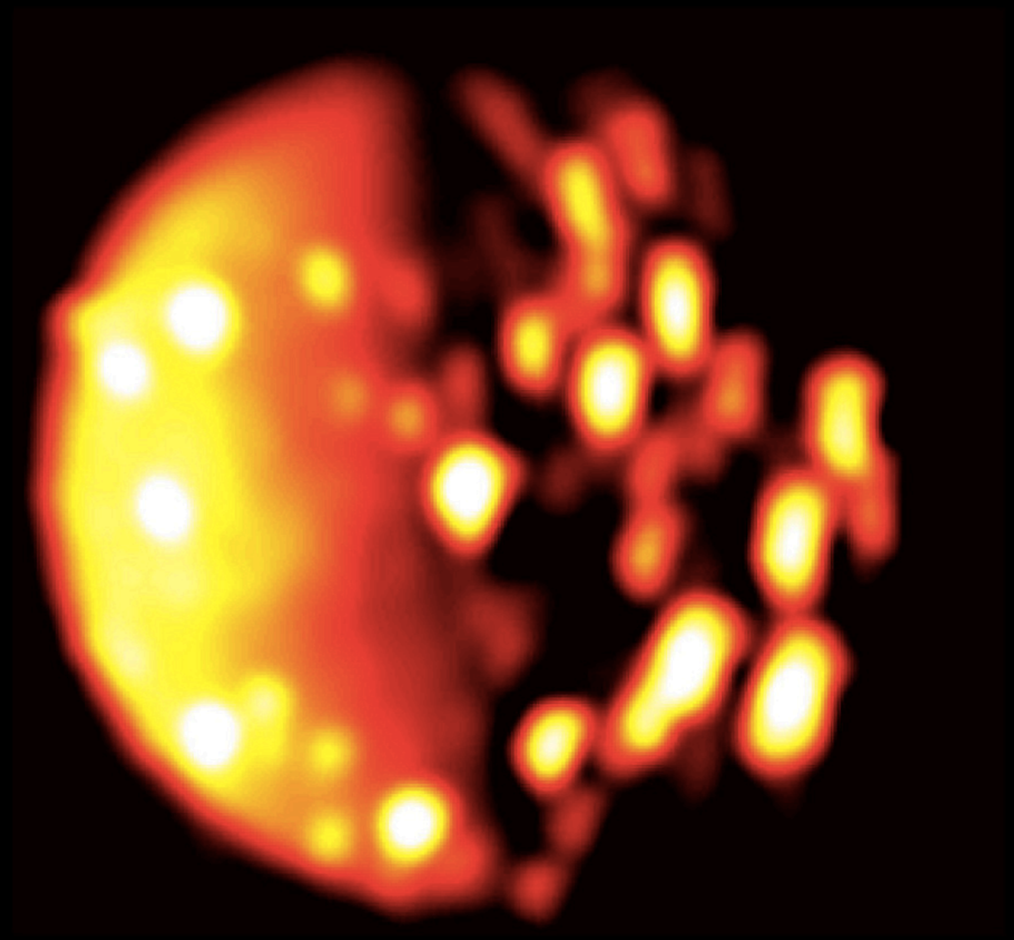

Questions?