Star-planet tidal interactions
Tidal interactions

- Why are tides important?
- A bit of theory
  - ★ Tides, the easy way
  - ★ Tides, the hard way…
- Some constraints brought by studying tides…
  - ★ Stars
  - ★ Planets
  - ★ Multi-planet systems
There are 2258 planets (over 5043, NASA Exoplanet Archive) with $P_{orb} < 10$ day.
Tidal interactions

Signs of star-planet interaction?

$P_{\text{orb}} = 1/2 P_{\star}$

$P_{\text{orb}} = P_{\star}$

[McQuillan + 13]
Habitable Zone

Only Habitable Zone planets available for atmospheric characterization!

[e.g. Fauchez+18, Lovis+17]

System around cool dwarfs
Tidal interactions
Planetary tide  
Stellar tide  
Distance  
Eccentricity  
Rotation  
Winds  
Seasons  
Tidal Heat Flux  
Energy  
Climate  

To be able to correctly identify a biosignature, we need to understand the system as a whole.
Tidal interactions

- Why are tides important?
- A bit of theory

★ Tides, the easy way
★ Tides, the hard way…
★ Some constraints brought by studying tides…
★ Stars
★ Planets
★ Multi-planet systems
A bit of theory

Ingredients

‣ At least 2 objects (could be star-planet, planet-satellite, star-star…)

‣ Extended objects (beyond newtonian point mass description)

‣ Some kind of dissipative process (e.g. thermal dissipation, viscous dissipation…)

‣ Objects shouldn’t be too far from each other

‣ Some time

No point mass allowed beyond this point
Let’s start simple

Tidal interaction appears when we consider that a body is not a point mass but extended.

Tides are a differential gravitational effect.
Tidal interaction appears when we consider that a body is not a point mass but extended.

Tidal field

**Acceleration** (in primary frame, which is accelerating at \( \mathbf{a}_P = \mathbf{G}(P) \)):

\[
\begin{align*}
    m\mathbf{a}_M|_P &= m\mathbf{G}(M) + \mathbf{F}' - m\mathbf{G}(P) \\
    m\mathbf{a}_M|_P &= m(\mathbf{G}(M) - \mathbf{G}(P)) + \mathbf{F}'
\end{align*}
\]

Tidal field \( \mathbf{C}(M) \)

Note that the primary frame here is not rotating, other terms would appear if we were to consider a co-rotating frame (rotation + Coriolis).

The tidal field \( \mathbf{C}(M) = \mathbf{G}(M) - \mathbf{G}(P) \) appears only when passing from an inertial frame to the primary-centered frame.
Tidal interaction appears when we consider that a body is not a point mass but extended.

The tidal field is given by:

$$C(M) = Gm_S \frac{r}{D^3} \left( C_r e_r + C_\theta e_\theta \right)$$

where

$$C_r = 3 \cos^2 \theta - 1$$
$$C_\theta = -\frac{3}{2} \sin 2\theta$$
Tidal interaction appears when we consider that a body is not a point mass but extended.
Simple case: coplanar, circular

Rotation frequency of primary $\Omega_p$

Orbital frequency of secondary $\propto 1/P$

Three different configurations:

$\star \Omega_p = n_s$

$\star \Omega_p < n_s$

$\star \Omega_p > n_s$
Simple case: coplanar, circular

\( \Omega_p = n_s \)

Orbital distance of secondary \( a_s = r_c \)

Corotation radius

Earth-Moon system

It’s the tidal equilibrium state!
Simple case: coplanar, circular

- ★ $\Omega_p < \Omega_s$
- $a_s < r_c$

*Constant time lag* model ($\Delta t$): e.g. Mignard, 1979; Hut 1981

*Constant phase lag* model ($\mathcal{Q}$ or $Q'$): e.g. Goldreich 1963
Simple case: coplanar, circular

\[ \Omega_p < n_s \]
\[ a_s < r_c \]

\[ \Omega_\star < n_{HJ} \]

\[ L_{\text{tot}} = h_{\text{orb, secondary}} + L_{\text{rot, primary}} \]
\[ \propto \sqrt{a} \]
\[ \propto \Omega_p \]

Conservation of total angular momentum!

Hot Jupiter systems
Simple case: coplanar, circular

\[ \Omega_p > n_s \]

\[ a_s > r_c \]

Earth-Moon system
Simple case: coplanar, circular

Minimize the deformation/dissipation

- Synchronize the rotation
Not so simple case: eccentricity and obliquity
Not so simple case: eccentricity and obliquity
Not so simple case: eccentricity and obliquity

Minimize the deformation/dissipation

- Eccentricity $\rightarrow 0$
Not so simple case: eccentricity and obliquity

Minimize the deformation/dissipation

- Obliquity → 0
Not so simple case: rotation, eccentricity and obliquity

Minimize the deformation/dissipation

- Eccentricity $\rightarrow 0$
- Obliquity $\rightarrow 0$
- Synchronize the rotation

$\Omega_* = \Omega_p = n$
Is an equilibrium always possible?

\[ \Omega_p < n_s \quad \Rightarrow \quad \Omega_p \quad n_s \]

\[ \Omega_p > n_s \quad \Rightarrow \quad \Omega_p \quad n_s \]

Can the system reach an equilibrium \((\Omega_p = n_s)\)?

Yes!

No…
Is an equilibrium always possible?

\[ \begin{align*}
\star \Omega_p < n_s & \quad \rightarrow \quad \Omega_p \\
\star \Omega_p > n_s & \quad \rightarrow \quad \Omega_p
\end{align*} \]

Can the system reach an equilibrium (\(\Omega_p = n_s\))? 

L is the total angular momentum (conserved quantity) 

h is the orbital angular momentum

Equilibrium exists if \(L > L_{\text{crit}}\) (which depends on the masses and moments of inertia)

Stable equilibrium reachable if \(h > 3/4 L\)
Is an equilibrium always possible?

- ★ $\Omega_p < n_s$ → $\Omega_p$ $\leftarrow n_s$
- ★ $\Omega_p > n_s$ → $\Omega_p$ $\leftarrow n_s$

Can the system reach an equilibrium ($\Omega_p = n_s$)?

Earth-Moon system today ($a = 60 \text{ } R_\oplus, P_\oplus = 24 \text{ } \text{day}$)

Equilibrium at $a \approx 90 \text{ } R_\oplus$ and $P_\oplus \approx 52 \text{ } \text{day}$

[Counselman, 1973]
Stellar tide

Planetary tide
Stellar tide

- Planet inside corotation $\rightarrow$ planet migrates inward

- Planet outside corotation $\rightarrow$ planet migrates outward

- Eccentricity decreases

- Inclination of planetary orbit decreases

- Timescales depend on stellar radius and the stellar dissipation
Stellar tide

- Planet inside corotation $\rightarrow$ planet migrates inward
- Planet outside corotation $\rightarrow$ planet migrates outward
- Eccentricity decreases
- Inclination of planetary orbit decreases
- Timescales depend on stellar radius and the stellar dissipation

In many articles, you might find the tidal quality factor $Q$ (or time lag $\Delta t$)

Low $Q$ (high $\Delta t$) means a fast evolution
High $Q$ (low $\Delta t$) means a slow evolution

Stars: $Q_\star \approx 10^5 - 10^8$ [Penev, 2018]
Planetary tide

- **Circular** orbit: quick synchronization
- **Eccentric** orbit: quick pseudo-synchronization / spin-orbit resonance

**Weakly viscous fluid approximation**
- e.g. constant time lag model [e.g. Hut 1981]

Eccentricity = 0 → **Synchronization**
Eccentricity ≠ 0 → **Pseudo-synchronization**

**Anelastic material approximation**
- e.g. Andrade rheology [e.g. Efroimsky+, Makarov+13]

Eccentricity = 0 → **Synchronization**
Eccentricity ≠ 0 → **Spin-orbit resonance**

Ex: Mercury has $P_{\text{rot}} = \frac{2}{3} P_{\text{orb}}$
Planetary tide

・Circular orbit: quick synchronization

・Eccentric orbit: quick pseudo-synchronization / spin-orbit resonance

・Obliquity of planet decreases

・Eccentricity of planet decreases

・Planet migrates inward

・Due to deformation, planet generates heat

・Timescales depend on planetary radius and the planetary dissipation
Planetary tide

- Circular orbit: quick synchronization
- Eccentric orbit: quick pseudo-synchronization / spin-orbit resonance
- Obliquity of planet decreases
- Eccentricity of planet decreases
- Planet migrates inward
- Due to deformation, planet generates heat
- Timescales depend on planetary radius and the planetary dissipation

In many articles, you might find \( Q \) (or \( \Delta t \))

Low \( Q \) (high \( \Delta t \)) means a fast evolution
High \( Q \) (low \( \Delta t \)) means a slow evolution

Stars: \( Q_\star \approx 10^5 - 10^8 \) [Penev, 2018]
Earth: \( Q_\oplus \approx 12 \) (\( \Delta t = 638 \) s) [Goldreich & Soter 1966; Neron de Surgy & Laskar 97]
Jupiter: \( Q_{jup} \approx 3 \times 10^4 \) [e.g., Lainey+2009, for Io’s frequency]
Before we go more into details...

Any questions?
Tidal interactions

- Why are tides important?
- A bit of theory
  - Tides, the easy way
  - Tides, the hard way… 😓
- Some constraints brought by studying tides…
  - Stars
  - Planets
  - Multi-planet systems
Tidal interactions

Tidal force perturbs the hydrostatic balance. And this results in:

- A mass redistribution
- Perturbations of the gravitational potential
There are two components to tides:

- The equilibrium tide
  - Large-scale circulation resulting from the hydrostatic adjustment to the tidal perturbation

- The dynamical tide
  - Fluid (elastic) eigenmodes of oscillations of the distorted body
Let us consider a spherical body $C$ (the primary) of mass $M$ on presence of a second body $S$ (secondary) of mass $m$.

How does the primary **adjust its shape** so that all forces are balanced?
i.e. so that it is in **hydrostatic equilibrium**?

**Hydrostatic equilibrium**

\[
0 = -\frac{1}{\rho} \nabla P - \nabla \Phi
\]

- $P$: Pressure gradient force
- $\Phi$: Gravitational potential

---

Let us consider a spherical body $C$ (the primary) of mass $M$ on presence of a second body $S$ (secondary) of mass $m$.
You can show that the shape of the deformed primary is close to a shape for which surfaces of constant $\rho$ coincides with surfaces of constant $\mathcal{V}$.

- Need to calculate $\mathcal{V}$
Tidal theory: perturbing potential

Let's calculate the perturbing tidal potential
Let us consider $\mathbf{r}$ the position of the secondary $S$ of mass $m$, and $\mathbf{r}'$ the position of a point $P'$ at the surface of the primary.

The gravitational potential $\mathcal{V}_S$ created by $S$ at the point $P'$ is given by:

$$\mathcal{V}_S(\mathbf{r}, \mathbf{r}') = -\frac{G m}{\| \mathbf{r} - \mathbf{r}' \|}$$

$$= -\frac{G m}{\sqrt{r^2 + r'^2 - 2(\mathbf{r} \cdot \mathbf{r}')}}$$
The gravitational potential $V$ created by $P$ at the point $P'$ is given by:

$$V(r, r') = -\frac{Gm}{\sqrt{r^2 + r'^2 - 2(r \cdot r')}}$$

The term $1/\sqrt{r^2 + r'^2 - 2(r \cdot r')}$ can be rewritten using Legendre polynomials:

$$V(r, r') = -\frac{Gm}{r} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \phi)$$

$$= \sum_{l=0}^{\infty} V_l(r, r')$$

where $\phi$ is the angle between $r$ and $r'$. 

Tidal theory: perturbing potential

The diagram illustrates the gravitational field created by a primary body $M$ and a secondary body $S$. The angle $\phi$ between the position vectors $r$ and $r'$ is shown, along with the perturbing potential $V_l(r, r')$. The potential is a series expansion involving Legendre polynomials, allowing for the calculation of tidal effects induced by the gravitational interaction.
The gravitational potential $\mathcal{V}$ created by $P$ at the point $P'$ is given by:

$$\mathcal{V}(\mathbf{r}, \mathbf{r}') = \sum_{l=0}^{\infty} V_l(\mathbf{r}, \mathbf{r}')$$

The first terms are given by:

$$V_0(\mathbf{r}, \mathbf{r}') = -\frac{\mathcal{G}m}{r}$$

$$V_1(\mathbf{r}, \mathbf{r}') = -\frac{\mathcal{G}m}{r} \cos \phi \frac{r'}{r}$$

$$V_2(\mathbf{r}, \mathbf{r}') = -\frac{\mathcal{G}m}{r} P_2(\cos \phi) \left( \frac{r'}{r} \right)^2 = -\frac{\mathcal{G}m}{r} \left( \frac{3}{2} \cos^2 \phi - 1 \right) \left( \frac{r'}{r} \right)^2$$

$V_0$ is constant in space so that $\mathbf{f} = -\nabla V_0$ is 0.

Responsible for Keplerian motion.

Tidal theory: perturbing potential
The tidal potential is therefore given by:

$$\mathcal{V}_{\text{tid}}(\mathbf{r}, \mathbf{r}') = -\frac{GM}{r} \sum_{l=2}^{\infty} P_l(\cos \phi) \left( \frac{r'}{r} \right)^l$$

In practice, we only consider the quadrupolar term $l = 2$

This is true if $r' \ll r$

The approximation is valid for $a > 5R_p$ and for small eccentricities [Mathis & Le Poncin-Lafitte 2009]
The tidal potential is therefore given by:

\[ \mathcal{V}_{\text{tid}}(\mathbf{r}, \mathbf{r}') = -\frac{Gm}{r} \sum_{l=2}^{\infty} P_l(\cos \phi) \left( \frac{r'}{r} \right)^l \]

In practice, we only consider the quadrupolar term \( l = 2 \)

This is true if \( r' \ll r \)

The approximation is valid for \( a > 5R_p \) and for small eccentricities

[Mathis & Le Poncin-Lafitte 2009]

We can express \( \cos \phi \) with the longitudes \( \varphi \) and \( \varphi' \) and colatitudes \( \theta \) and \( \theta' \) of \( P \) and \( P' \)

\[ \cos \phi = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi') \] (addition theorem)
The tidal potential is therefore given by:

\[
\mathcal{V}_{\text{tid}}(\mathbf{r}, \mathbf{r}') = -\frac{\mathcal{G} m}{r} \left( \frac{r'}{r} \right)^2 \times \left[ \frac{1}{2} (3 \cos^2 \theta' - 1) \frac{1}{2} (3 \cos^2 \theta - 1) + \frac{3}{4} \sin^2 \theta' \sin^2 \theta \cos \left( 2(\varphi - \varphi') \right) + \frac{3}{4} \sin 2\theta' \sin 2\theta \cos(\varphi - \varphi') \right]
\]

For the Earth-Moon case \((\Omega \gg n)\), we can see different components associated to different frequencies:

- The term in \(\cos^2 \theta = 1/2(1 + \cos 2\theta)\) varies with frequency \(2n\): it's the fortnightly tide
- The term in \(\cos(\varphi - \varphi')\) varies with a frequency of \(\Omega - n \approx \Omega\): it's the diurnal tide
- The term in \(\cos(2(\varphi - \varphi'))\) varies with a frequency of \(2(\Omega - n) \approx 2\Omega\): it's the semi-diurnal tide
The term in $\cos(2(\varphi - \varphi'))$ varies with a frequency of $2(\Omega - n) \approx 2\Omega$: it's the semi-diurnal tide.

2 high tides in one day
A convenient way of writing the potential comes from Kaula \cite{1962, 1964}: It allows to transform the coordinates of the secondary \((r, \theta, \varphi)\) in useful dynamical parameters:

\[
\mathcal{V}_{\text{tid}}(r, r') = -\frac{Gm}{a} \sum_{l=2}^{\infty} \left( \frac{r'}{a} \right)^l \sum_{m=0}^{l} \frac{(l - m)!}{(l + m)!} (2 - \delta_{0m}) P_l^m(\cos \theta') \sum_{p=0}^{l} \sum_{q \in \mathbb{Z}} F_{lmp}(I) G_{lpq}(e)
\]

\[
\begin{align*}
\cos m\lambda' \left\{ \begin{array}{c}
\cos \\text{even} \\
\sin \\text{odd}
\end{array} \right. \\
\sin m\lambda' \left\{ \begin{array}{c}
\sin \\text{even} \\
-\cos \\text{odd}
\end{array} \right.
\end{align*}
\]

\[
\left[ \begin{array}{c}
\cos m\lambda' \left\{ \begin{array}{c}
\cos \\text{even} \\
\sin \\text{odd}
\end{array} \right. \\
\sin m\lambda' \left\{ \begin{array}{c}
\sin \\text{even} \\
-\cos \\text{odd}
\end{array} \right.
\end{array} \right] = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{p=0}^{l} \sum_{q \in \mathbb{Z}} V_{lmpq}
\]

\(\omega_{lmpq}\) are the frequencies of the forcing and are given by: \(\omega_{lmpq} = (l - 2p + q)n - m\Omega\)
A convenient way of writing the potential comes from Kaula [1962, 1964]: It allows to transform the coordinates of the secondary \((r, \theta, \varphi)\) in useful dynamical parameters:

- semi-major axis \(a\),
- eccentricity \(e\),
- inclination \(I\),
- argument of periastron \(\omega^*\),
- argument of ascending node \(\Omega^*\)

\(\omega_{impq}\) are the frequencies of the forcing and are given by: \(\omega_{impq} = (l - 2p + q)n - m\Omega\)

For \(l = 2\), a circular orbit, and a coplanar orbit, there is one excitation frequency given by:

\[\omega_{impq} = 2(n - \Omega)\]

(it's the semi-diurnal frequency)
A convenient way of writing the potential comes from Kaula [1962, 1964]:
It allows to transform the coordinates of the secondary \((r, \theta, \varphi)\) in useful dynamical parameters:

- semi-major axis \(a\),
- eccentricity \(e\),
- inclination \(I\),
- argument of periastron \(\omega^*\),
- argument of ascending node \(\Omega^*\)

\(\omega_{impq}\) are the frequencies of the forcing and are given by:

\[
\omega_{impq} = (l - 2p + q)n - m\Omega
\]

For \(l = 2\), a circular orbit, and a coplanar orbit, there is one excitation frequency given by:

\[
\omega_{impq} = 2(n - \Omega)
\]

(it's the semi-diurnal frequency)

For an eccentric orbit or for an inclined orbit, additional frequencies are excited
Tidal theory: deformed body potential

Let's calculate the potential created by deformed body
We use here the theory of Love:
The potential of the deformed body $\Phi$ at its surface is proportional to the corresponding component of the perturbing potential $\nu_{\text{tid}}$ at its surface [Love, 1911].

$$\Phi_{\text{deformed body}}(r = R_{\text{surface}}) = k_2(\omega) \times \Phi_{\text{secondary}}(r = R_{\text{surface}})$$

Response function
(depending on properties of primary)
The tidal response can be divided into two components:

- Effects associated by the non-spherical shape of the distorted body: instantaneous response (non-dissipative)
  
  Responsible for orbital precession

\[k_i(\omega_{lmpq}) = \text{Re}k_i(\omega_{lmpq}) + i\ \text{Im}k_i(\omega_{lmpq})\]
The tidal response can be divided into two components:

- Effects associated by the non-spherical shape of the distorted body: instantaneous response (non-dissipative)
  - Responsible for orbital precession
- Effects associated by the viscosity/rheology of the distorted body: delayed response (dissipative)
  - Responsible for orbital and rotational evolution
Natural way to express dissipation

Phase lag

$$\delta_l(\omega_{lmpq}) = e_l(\omega_{lmpq})/2$$

Geometrical lag angle

$$k_l(\omega_{lmpq}) = |k_l(\omega_{lmpq})| e^{-ie_l(\omega_{lmpq})}$$

Tidal theory: deformed body potential
Tidal theory: deformed body potential

Using one tidal quality factor $Q$ is equivalent of doing many approximations: in particular the phase lag $\epsilon_2(\omega) = \epsilon = $ cst [Goldreich 1963]

Using one time lag $\Delta t$ is equivalent of doing many approximations: in particular the phase lag $\epsilon_2(\omega) \propto \omega$ [Darwin 1879]

The phase lag has a smooth dependency in the excitation frequency

Equilibrium tide

⇒ Appropriate for objects made of weakly viscous fluid
Tidal interactions

There are two components to tides:

- The equilibrium tide
  Large-scale circulation arising from the hydrostatic adjustment to the tidal perturbation
- The dynamical tide
  Fluid (elastic) eigenmodes of oscillations of the distorted body

Tidal response depends on the properties of the extended body
There are two components to tides:

- **The equilibrium tide**
  
  Large-scale circulation resulting from the hydrostatic adjustment to the tidal perturbation

- **The dynamical tide**
  
  Fluid (elastic) eigenmodes of oscillations of the distorted body
Dynamical tide

Turbulent friction (viscous diss.)
i.e., convection zone of Sun-like stars/gas giants (dynamics driven by Coriolis acc.)

Heat diffusion (thermal diss.)
i.e., radiative zone of Sun-like stars/oceans (dynamics driven by buoyancy acc.)

0
Alfvén waves
Gravity waves
Inertial waves
2Ω is the inertial frequency
2Ω is the Alfvén frequency

ω
ωA

N

fL

Adapted from Mathis & Remus 2013
Dynamical tide

Excitation by each Fourier component of the potential

$\omega_A$ is the Alfvén frequency

$2\Omega$ is the inertial frequency

$N$ is the Brunt-Väisälä frequency (or buoyancy frequency)

$f_L$ is the Lamb’s frequency

Adapted from Mathis & Remus 2013
Tidal interactions

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Stellar tide

Stellar parameters

Tidal response

Orbital/rotational evolution

Internal structure
Stellar tide

Observational constraints

- **Meibom & Mathieu [2005]** used the tidal circularization of binaries in an open cluster to estimate \( Q'_* \approx 10^6 \)

- **Jackson+2008** used the tidal circularization of a small sample of exoplanets and found a best fitting value of \( Q'_* = 10^{5.5} \) (they also fit a planetary \( Q_p \))

- **Collier Cameron & Jardine 2018** used the orbital distance distribution of HJs to calculate \( \log_{10} Q'_* = 8.26 \pm 0.14 \) for the equilibrium tide regime, but a smaller value of \( \log_{10} Q'_* = 7.3 \pm 0.4 \) for the dynamical tide regime

- Using the fact that inward migration of a massive planet leads to a spin up of the star, **Carone & Pätzold [2007]** analyzed the system OGLE-TR-56 and found \( Q'_* > 2 \times 10^7 \)

**Penev+18** also used this phenomenon for a statistical study of HJ hosts and find that \( Q'_* \) depends on the forcing frequency
Stellar tide

Observational constraints

- For planetary systems close to the edge of tidal disruption, it could be possible to measure the transit timing variation due to the inward migration. Birkby+ [2014] showed that a baseline of a few years is necessary.

- Recently Yee+ [2020] used the transit timing variation of the WASP-12b system (29 ± 2 ms yr⁻¹) to estimate $Q'_* = 1.8 \times 10^5$. 

![Graph showing transit time shifts after 10 years assuming $Q'_* = 10^6$ (circular orbits)](image-url)

Transit time shift after 10 years assuming $Q'_* = 10^6$ (circular orbits)
Stellar tide: equilibrium tide

Velocity field of the equilibrium tide

Dissipation

Friction applied by turbulence

Diffusion of heat

Dissipation in convective region is higher [Zahn 1977]
Stellar tide: equilibrium tide

Velocity field of the equilibrium tide

\[ \nu_t \text{ (turbulent viscosity)} \]

\[ \frac{k_2}{Q} = \left| \frac{\Phi_{II}}{U} \right| \]

\[ \frac{t_{core}}{P_{tide}} \]

life span of the convective elements

[Zahn 1966a; Remus+12]
Stellar tide: dynamical tide

Tidal inertial waves in the convective zone

Equilibrium tide

\[ P_{\text{orb}} = \frac{1}{2} P\star \]
Stellar tide: dynamical tide

Tidal inertial waves in the convective zone

\[ E_{k} \]

[Guennel+16]

Inertial waves

\[ P_{\text{orb}} > 1/2 \ P_{*} \]

\[ P_{*} = 10 \text{ days} \]

Sun, with a rotation period of 10 days [Ogilvie & Lin, 07]
Stellar tide: dynamical tide

Tidal inertial waves in the convective zone

Equ. Tide

Inertial waves

$P_\text{orb} > 1/2 \ P_*$

$P_* = 10 \text{ days}$

More peaks
Higher dissipation

Average value of dissipation depends on structural parameters and rotation [Ogilvie 2013]

Equ. Tide

$P_* = 3 \text{ days}$

Sun, with a rotation period of 10 days [Ogilvie & Lin, 07]

Sun, with a rotation period of 3 days [Ogilvie & Lin, 07]
Tidal inertial waves in the convective zone

Average value of dissipation depends on structural parameters and rotation

\[ \alpha = \frac{R_c}{R_*} \]

\[ \beta = \frac{M_c}{M_*} \]
Stellar tide: dynamical tide

Tidal inertial waves in the convective zone

Average value of dissipation depends on structural parameters and rotation

\[ \alpha = \frac{R_c}{R_*}, \quad \beta = \frac{M_c}{M_*}, \]

\[ \text{Im} k_2 = f(\alpha, \beta)g(\Omega) \]

Only structural part (\( \Omega = \text{cst} \))

Structural part + rotational part

\[ \Omega = \text{cst} \]

\( Q \) reaches a plateau while the eon the Henyey phase, the dissipation (equivalent quality factor) star contracts and its core develops, the tidal dissipation (equivalent quality factor) since (i.e. by the internal structure). During the Hayashi phase, as the behavior of the dissipation, on the MS phase, is drastically orders of magnitude toward lower intensity as (i) that the tidal dissipation is lower, on the PMS, by about two main e

\( Q_{0s} = \frac{Q_{0}}{\Omega_{0}} = \frac{Q}{\Omega} \leq 3 \quad (2 < \Omega_{0} < 2.2) \).

As a consequence, the tidal dissipation (equivalent quality factor) is controlled by the evolution of the surface angular velocity and thus by the extraction of angular momentum (see Sect. 2.2). As a consequence, the tidal dissipation (equivalent quality factor) decreases (increases) as the star slightly expands. Just before the ZAMS, the dissipation (equivalent quality factor) continuously decreases (increases) towards the TAMS. During the MS phase, and as pointed out above, both the efficiency of the braking of the magnetic field that observationally appears around Ro this magnetic saturation is to reduce the e

During that phase, while the temperature slightly decreases (increases) at quasi constant e

The evolution of the normalized tidal dissipation follows the law (see Kawaler 1988). During that phase, while the temperature slightly decreases (increases) at quasi constant e

Even if the rotation rate is evolving during the PMS phase, the dissipation (equivalent quality factor) continuously decreases (increases) towards the TAMS. At that point, the internal structure stops to control the evolution of the MS phase. At that point, the internal structure stops to control the evolution of the MS phase. At that point, the internal structure stops to control the evolution of the MS phase.
Stellar tide: dynamical tide

Tidal response → Orbital/rotational evolution

Tidal inertial waves in the convective zone

P_{\text{orb}} = P_*

Age - t_{\text{init}} (yr)

Semi-major axis (AU)

Outward migration

Inward migration

Equilibrium tide model

Dynamical tide model

[Bolmont\&Mathis 16]

10^2 10^4 10^6 10^8 10^{10}
Stellar tide: dynamical tide

Tidal response $\rightarrow$ Orbital/rotational evolution

Tidal inertial waves in the convective zone

$P_{\text{orb}} = \frac{1}{2} P_{\star}$

![Graph showing age vs. semi-major axis with dynamical and equilibrium tide models](image)

[Bolmont & Mathis 16]
Stellar tide: dynamical tide

Tidal response ➔ Orbital/rotational evolution

Tidal inertial waves in the convective zone

\[ P_{\text{orb}} = 1/2 \, P_* \]

[Equilibrium tide model]

[Tidal response model]

\[ P_{\text{orb}} = P_* \]

\[ \text{Semi-major axis (AU)} \]

\[ \text{Age - } t_{\text{init}} \, (\text{yr}) \]

[Bolmont&Mathis 16]

Tidal inertial waves in the convective zone

Orbital/rotational evolution
Stellar tide: dynamical tide

Tidal inertial waves in the convective zone

Equilibrium tide model
Dynamical tide model

Tidal response → Orbital/rotational evolution

\[ P_{\text{orb}} = \frac{1}{2} P_{\bullet} \]

Semi-major axis (AU)

Age - \( t_{\text{init}} \) (yr)

[Bohlmont & Mathis 16]
Stellar tide: dynamical tide

Tidal response ➡ Orbital/rotational evolution

Effect of tidal inertial waves in the convective envelope of Sun-like stars: Bolmont & Mathis 2016, Gallet+17, Benbakoura+19, Ahuir+21a…

.shapes the architecture of the young planetary systems

Effect of tidal gravity waves in the radiative zone of Sun-like stars: Ahuir+21b, Lazovik+21…
Stellar tide: dynamical tide

Tidal response → Orbital/rotational evolution

Effect of tidal inertial waves in the convective envelope of Sun-like stars:
Bolmont & Mathis 2016, Gallet+17, Benbakoura+19, Ahuir+21 a...

- Shapes the architecture of the young planetary systems

Effect of tidal gravity waves in the radiative zone of Sun-like stars:
Ahuir+21 b, Lazovik+21...

- Shapes the architecture of the “old” planetary systems

[Image]

Figure 3.

Vorb = Rorb = ⌦y. a. lazovik

Illustrates the secular evolution of a star-planet system correspond bins.
Black solid line corresponds to the best fit, expressed by

3 ORBITAL EVOLUTION

Represent bins, black solid line corresponds to the best fit, expressed by

- Corotation
- Dyn Tides (Gravity waves)
- Equilibrium Tide
- Inertial waves

Shapes the architecture of the young planetary systems

Shapes the architecture of the “old” planetary systems
A planetary magnetic field that can reach $10 \, \text{G}$. Hence, we adopt (situated near the Roche limit of a
rocky super-Earth), in the case discussed in Sect.
3. The thick curves correspond to our reference
case with a planetary magnetic field $B_p = 10 \, \text{G}$. This leads to a delay of around $100 \, \text{Myr}$
and an increase in the dipolar torque by a factor of 3.6, which has a
significant influence on the semi-major axis of the planet during
the secular evolution, manifesting as an increase in the initial semi-

major axis by a factor of two and an increase in the initial stellar
mass by a factor of five.

This leads to a stronger mass dependency of the magnetic torque
in the case of close-in planets with a synchronous rotation and a
rotation period by a factor of five. In such a scenario, according to the
instance, one can assume that the magnetic field of the planet
finds this effect to be negligible and choose to consider a con-
stant surface magnetic field scenario, the tidal torque is more
sensitive to planetary mass than the SPMI torque. Hence, we
consider behaves in the same way as what is observed in the
most magnetized hot Neptune (situated near the Roche limit of a
planet). While the maximal field strength in the Solar System to assess
the moment of rocky planets from dynamo models, which leads to a
dependency in the case of neptunian planets. However, as in the
case of close-in planets with a synchronous rotation and a
rotation period by a factor of five.

The magnetic torque then scales as

$$\propto \frac{B_p^2 R_d^4}{M_p^6}$$

from the
relationships (Eq. (35)), a higher planetary magnetic field leads to more
significant migration. Such a trend is illustrated in Fig.
9. From the
Chen & Kipping
2017
2017

87

K star orbited by a magnetized hot Neptune [Ahuir+21]
Tidal interactions

- Why are tides important?
- A bit of theory
  - ★ Tides, the easy way
  - ★ Tides, the hard way…
  - ★ Some constraints brought by studying tides…
  - ★ Stars
  - ★ Planets
  - ★ Multi-planet systems
Planetary tide

- Radius
- Mass
- Tidal response
- Orbital/rotational evolution
- Potential composition and internal structure [Sotin+07]
- Rheology
Planetary tide: rocky planets/cores

Rheology

Anelastic Solid or Standard Linear Solid, has features of both spring. While the Voigt–Kelvin model is instructive as a sub-

Mathematically, a spring damper model is represented by and dampers that model viscous creep and elastic rebound. The four basic models and parameters for the rheologies of grains. Grain boundary slip occurs on a shorter relaxation timescale and is recoverable, as represented by the Voigt–Kelvin model.

The four models and parameters for the rheologies — Correia & Laskar (2014) and Efroimsky — are resolved within a corresponding notation.

As discussed by Zener (1948), we ultimately find it poorly suited to short-
tidal evolution. This threshold is about 1 yr for the solid Earth, this threshold is about 1 yr. Second, either the condition µ ≥ 2σ/R₄

The values of ϵ /ζ = 1 and the 4/ζτ are calculated using Equation (12).

Using the geometry and parameters of each model from Table 1, estimates of J = 10^{15} /µσ and the quality factor K(ΔΩ) ≈ 4yQ. The quality factor K(ΔΩ) ≈ 4yQ.

The dependences of the tidal evolution rate and of the tidal response, followed by strain limited relaxation. It will not take 

true in a Voigt–Kelvin model, where rapid forced strain cycles can drive the dashpot to approach infinite work. Mainly for this reason, we find the Voigt–Kelvin model unsuited to tidal cases, can drive the dashpot to approach infinite work. Mainly for this reason, we find the Voigt–Kelvin model unsuited to tidal cases.

Second, either the condition µ ≥ 2σ/R₄ = 2σ/(R₄(1 + M /R₄M)) where M /R₄M can drive the dashpot to approach infinite work. Mainly for this reason, we find the Voigt–Kelvin model unsuited to tidal cases.

Suggested that the Burgers body is a more appropriate model of fo a parameter model, or Burgers body, allows them model -

Larger Superearth

Andrade

Iapetus

Mars

Earth

Superearth

Larger Superearth

[Makarov+18]

[Makarov+18]

[Efroimsky+12]
Planetary tide: rocky planets/cores

Tidal response  ➔  Orbital/rotational evolution

Weakly viscous fluid approximation
- e.g. constant time lag model [e.g. Hut 1981]

Eccentricity = 0 ➔ Synchronization
Eccentricity ≠ 0 ➔ Pseudo-synchronization

Anelastic material approximation
- e.g. Andrade rheology [e.g. Efroimsky+, Makarov+13]

Eccentricity = 0 ➔ Synchronization
Eccentricity ≠ 0 ➔ Spin-orbit resonance

Ex: Mercury has $P_{\text{rot}} = \frac{2}{3} P_{\text{orb}}$
Dynamical evidence for Phobos and Deimos as remnants of a disrupted common progenitor.
The major component of tidal dissipation for the Earth comes from the ocean (especially in shallow regions). Without oceans the overall dissipation of the Earth would be 1/10th of what it is today.
Planetary tide: rocky planets/liquid layers

Tidal response  Orbital/rotational evolution

Evolution of Phobos’s semi-major axis

Rocky body

\[ Q \propto \omega^\alpha \]

\[ \text{CPL } Q = \text{cst} \]

\[ \text{CTL } \Delta t = \text{cst} \]

Smooth evolution

[Efroimsky & Lainey 2007]

Evolution of Phobos’s semi-major axis

Fluid body

\[ \text{constant } Q \]

\[ \text{peaks model} \]

Erratic evolution

[Auclair-Desrotour+ 14]
Planetary tide: rocky planets/liquid layers

Earth-Moon system

[Farhat+22]

Complex evolution with multiple crossings of resonances

Reproduces well the data!
Planetary tide: gas giants

Constraints from the Solar System

Dissipation depends on frequency

Dissipation stronger than what CPL model predicts

Saturn

Enceladus

Tethys

Dione

Rhea

Saturn's $k_n Q$

Highest possible value from Sinclair (1983)

[Tidal frequency $2(1-n)$ in rad/sec]

[Lainey+09, 12, 17]
Tidal interactions

- Why are tides important?
- A bit of theory
  - ★ Tides, the easy way
  - ★ Tides, the hard way…
- Some constraints brought by studying tides…
  - ★ Stars
  - ★ Planets
  - ★ Multi-planet systems
Tides in multiplanet systems
Tides in N-body systems

Tidal evolution for multiple planet-systems:

If equilibrium is possible \[ L_{\text{orb}} > \frac{3}{4} \left( L_{\text{orb}} + L_{\text{rot}} \right) \], then:

- Eccentricity = 0
- Planetary spin and orbital angular momentum aligned
- Planetary spin synchronized
If equilibrium is possible \( L_{\text{orb}} > 3/4 (L_{\text{orb}} + L_{\text{rot}}) \), Hut, 1980, then:

- Eccentricity = 0
- Planetary spin and orbital angular momentum aligned
- Planetary spin synchronized

\[ \rightarrow \] Eccentricity reaches an equilibrium between tidal damping and planet-planet excitation

\[ \rightarrow \] Obliquity reaches an equilibrium

\[ \rightarrow \] Rotation depends on eccentricity and is influenced by planet-planet excitation

Tidal evolution for multiple planet-systems:
Tides and stability

The assessment of **stability** depends on whether **tides** are taken into account.

---

**Kepler-62 System**

Star:

- $M_{\star} = 0.69 \, M_{\odot}$

5 planets:

- $0.54 < R_p/R_{\odot} < 1.95$
- $0.05 < a/AU < 0.72$

Borucki+13
Tides and stability

The assessment of **stability** depends on whether **tides** are taken into account.
Constraining the eccentricity

Star:
$M_★ = 0.95 \, M_⊙$

5 planets:
$0.015 < a/AU < 5.4$

Demory+11

55 Cancri
Constraining the eccentricity

For an Earth-like dissipation

Equilibrium between planet-planet excitation and tidal damping

Orbital parameters from Dawson & Fabrycky (2010)

Eccentricity of planet e should be lower than 0.002

Bolmont et al. 2013
Tides in N-body systems: TRAPPIST-1

Star:
\[ M_\star = 0.09 \, M_\odot \]

5 planets:
\[ 0.76 < \frac{R_p}{R_\odot} < 1.1 \]
\[ 0.01 < \frac{a}{\text{AU}} < 0.06 \]

Gillon+16,17
Constraining the eccentricity

Dissipation factor
- $\sigma_p = 0.1 \sigma_\oplus$
- $\sigma_p = 1 \sigma_\oplus$
- $\sigma_p = 10 \sigma_\oplus$

All planets should have eccentricities lower than 0.01.
Planets b & c are likely to have eccentricities lower than 0.001.
Constraining the rotation

The rotation evolves very fast!

In ~200,000 yr:
- **Obliquity** damped
- **Rotation** pseudo-synchronized
Constraining the rotation

- Impact on the **climate** of the planets

---

The rotation evolves very fast!

In \(\approx 200,000\) yr:
- **Obliquity** damped
- **Rotation** pseudo-synchronized
Constraining the tidal heat flux

Non zero eccentricity and obliquity → Tidal heat flux
Constraining the tidal heat flux

Tidal heating? Significant or not?

- Tidal heat flux using CTL model [from N-body simulations with tides; Turbet+18]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tb</th>
<th>Tc</th>
<th>Td</th>
<th>Tc</th>
<th>Tf</th>
<th>Tg</th>
<th>Th</th>
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<tbody>
<tr>
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<tr>
<td>ecc max (×10^{-3})</td>
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<td>1.2</td>
<td>5.9</td>
<td>8.3</td>
<td>9.7</td>
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<tr>
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<td>0.38</td>
<td>0.12</td>
<td>0.02</td>
<td>&lt;10^{-3}</td>
<td>&lt;10^{-4}</td>
<td>W m^{-2}</td>
</tr>
</tbody>
</table>

≈ Earth’s heat flux [Pollack+93]

> Io’s tidal heat flux [Spencer+00]

- Maximum tidal heating for uniform planets and Maxwell rheology [Makarov+18]

- Tidal heat flux using model for “uniform” planets and Maxwell rheology [uniform viscosity and rigidity based on each planet’s composition; Barr+18]

- Tidal heat flux using model with multi-layer bodies and Andrade’s rheology [Bolmont+20]
Tidal heat flux and habitability

Non resonant system

Resonance 2:1

Brown dwarf and 3 Earth-like planets [Bolmont 2018]
Tidal heat flux and habitability

Non resonant system

Habitable zone

Resonance 2:1

Orbital distance (AU)

Eccentricity

Time (years)

Also discussed in Barnes+10 for planets around M-dwarfs

Bolmont 18
Tidal heat flux and habitability

Non resonant system

Resonance 2:1

Also discussed in Barnes+10 for planets around M-dwarfs
Questions?