## Building theoretical molecular spectra

Role of ab initio calculations \& construction of line lists

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## Motivations of this lecture

## An Overview of Molecular Opacities

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${ }^{2}$ Also, University of Waterloo, Waterloo, Ontario, Canada
$\checkmark$ Construction of reliable \& complete spectroscopic line lists
$\Rightarrow$ Earth atmosphere
$\Rightarrow$ Planetological (Titan,..)
$\rightarrow$ Astrophysical (hot bodies,...)

Molecules are ubiquitous in the Universe and are found in diverse sources including diffuse clouds, dark clouds, giant molecular clouds, planetary nebulae, circumstellar envelopes, stellar and brown dwarf atmospheres, protoplanetary disks, comets, supernovas, planetary and exoplanetary atmospheres, and galaxies (Bernath 2009). Except


## Aim of this lecture: going inside the molecule

Describe the construction of molecular databases, step-by-step
Show that quantum-mechanical calculations are very demanding, even for small molecules
Not a black-box procedure, both for solving electron and nuclear motion equations
Show that building spectra requires highly-optimized computational methods

We will see that solving a quantum chemical or quantum mechanical problem will consist in making a series of approximations

Find the good compromise between accuracy and feasibility

## A mandatory step before the line list...

This lecture

(*) Latin « ab initio » = « from the beginning » = no prior experimental knowledge

## Role of $a b$ initio in modern spectroscopy

$a b$ initio is strongly involved in spectroscopic databases for historical molecules
Diatomics, $\mathrm{H}_{2} \mathrm{O}, \mathrm{NH}_{3}$ by UCL London
Ozone, methane, phosphine, etc. in HITRAN/GEISA
Future updates in databases (ethane, $\mathrm{SF}_{6}, \mathrm{CF}_{4}, \mathrm{C}_{2} \mathrm{H}_{4}$, formaldehyde, etc. ?)

## Role of $a b$ initio in modern spectroscopy

HR spectra analyses are completely revisited : $a b$ initio intensities may replace the experimental measurements in some cases (better when many overlaped lines difficult to extract, for dense spectra like those of hot or heavy molecules)


"change of paradigm"<br>in high-resolution infrared spectroscopy?

## Available databases

|  | (I) <br> Experimental, empirical or both | (II) <br> From first-principles (ab initio + variational) |  |
| :---: | :---: | :---: | :---: |
|  | HITRAN <br> GEISA <br> WKLMC <br> PNNL <br> VAMDC | ExoMol (UCL London) NASA Aims TheoReTS (Reims - Tomsk) <br> More recently, in Stuttgart (Rauhut et al. |  |
| Designed for Room T <br> Limited for high T \& Wnb <br> Few HBs Accurate in position | STDS <br> SMPO | How to build such line lists?? | Way more complete <br> Designed for low \& high T <br> Lower accuracy in position |

# « Numerical recipe » for building a 

## vibration-rotation spectrum

## from $a b$ initio surfaces <br> (not unique - there exists other ones)

## Ingredients

(1) $a b$ initio PES \& DMS

Commercial codes (Molpro, etc.)
(resolution of the electronic SE: domain of quantum chemistry)
(2) Resolution of the Schrödinger equation for the nuclear motions.

Home-made computer codes

## The molecular Hamiltonian

For a molecules composed of $N$ nuclei \& $n$ electrons

$$
\begin{array}{r}
H=-\sum_{i=1}^{n} \frac{\hbar^{2}}{2 m_{e}} \nabla_{e_{i}}^{2}-\sum_{I=1}^{N} \frac{\hbar^{2}}{2 m_{N_{I}}} \nabla_{N_{I}}^{2}+\sum_{i<j}^{n} \frac{e^{2}}{4 \pi \epsilon_{0} r_{i j}}+\sum_{I<J}^{N} \frac{Z_{I} Z_{J} e^{2}}{4 \pi \epsilon_{0} r_{I J}}-\sum_{i}^{n} \sum_{I}^{N} \frac{Z_{I} e^{2}}{4 \pi \epsilon_{0} r_{I i}} \\
\begin{array}{c}
\text { Kinetic } \\
\text { (electron) }
\end{array} \\
\text { (nuclei })
\end{array} \begin{gathered}
\text { Coulomb } \\
(e-e)
\end{gathered} \underset{(n-n)}{\text { Coulomb }} \begin{gathered}
\text { Coulomb }
\end{gathered}
$$

No chance to solve the Schrödinger equation exactly!

Approximations are thus required

## The Born-Oppenheimer approximation

In the Schrödinger equation, nuclear coordinates are parameters and the nuclei are held fixed
$\Rightarrow$ Nuclei are 1800 times heavier than an electron
$\Rightarrow$ Separation of the wavefunctions
$\rightarrow$ The Schrödinger equation is solved in sequence


Step 1. Electronic structure calculation ( $a b$ initio methods $=$ developed by quantum chemists $=$ not detailed here )

$$
\left\{-\sum_{i=1}^{n} \frac{\hbar^{2}}{2 m_{e}} \nabla_{e_{i}}^{2}+\sum_{i<j}^{n} \frac{e^{2}}{4 \pi \epsilon_{0} r_{i j}}-\sum_{i}^{n} \sum_{I}^{N} \frac{Z_{I} e^{2}}{4 \pi \epsilon_{0} r_{I i}}\right\} \Psi_{e}(\mathbf{r} ; \mathbf{R})=E_{e}(\mathbf{R}) \Psi_{e}(\mathbf{r} ; \mathbf{R})
$$

Step 2. Nuclear dynamics

R : grid of points ${ }^{\sim} 10^{4}-10^{5}$ Polyatomic molecule

$$
\left\{-\sum_{I=1}^{N} \frac{\hbar^{2}}{2 m_{N_{I}}} \nabla_{N_{I}}^{2}+\sum_{I<J}^{N} \frac{Z_{I} Z_{J} e^{2}}{4 \pi \epsilon_{0} r_{I J}}+E_{e}(\mathbf{R})\right\} \Psi_{n}(\mathbf{R})=E \Psi(\mathbf{R})
$$

## The Born-Oppenheimer approximation

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Step 1. Electronic structure calculation (ab initio methods $=$ developed by quantum chemists $=$ not detailed here)
Use of commercial computer codes like MOLPRO, GAUSSIAN
Different levels of calculation
Quality will strongly depend on the number of electrons (degrees of freedom) - few days up to several months of calculation

## Step 2. Nuclear dynamics

Use of home-made computer codes (depending on the type of coordinates)

> Different levels of calculation
(Slow or fast) convergence of calculation will strongly depend on the number of atoms (degrees of freedom) - few hours up to few weeks of calculation

## Electronic structure calculations

MOLPRO, COLUMBUS, GAUSSIAN, etc.
Type of method / Type of orbital basis $\rightarrow$ for electron correlation, QED, DBOC, rel., etc


## The Born-Oppenheimer approximation

$\checkmark$ One of the most accurate approximations in quantum chemistry for quite isolated electronic states
$\checkmark$ But fails in presence of close (or degenerate) electronic states (open-shell molecules like ions or radicals) $\rightarrow$ coupling terms

Electronic Nonadiabatic Coupling Terms
and Conical Intersections


## At this stage...

$\checkmark$ Concept of (ab initio) Potentiel Energy (hyper)Surface introduced
$\checkmark$ Need to compute now the dynamics of the nuclei on this surface


Nuclear-motion, quantum mechanical calculations

## AIM: Calculated line lists

Line positions Line intensities Einstein Lower state E


## Step 1 - The nuclear Hamiltonian model

$H=T\left(X, Y, J_{\alpha}\right)+V(X)=\mathrm{KEO}+\quad=$ nuclear motion or initial or untransformed $H$
$X$ : nuclear coordinates e.g. curvilinear (valence, radau, jacobi, ...) or rectilinear (normal)
$Y$ : conjugate momenta
$J_{\alpha}$ : total angular momentum components (degree less or equal than 2)
$V(X)=E_{e}(X)$ : PES with empirical or $a b$ initio or empirically-refined $a b$ initio force constants

## A zero-order - HO+RR - approximation

$>$ In first approximation, the PES is a quadratic form
$\checkmark$ Potential = harmonic $\rightarrow$ harmonic oscillator for the vibration

$>$ We can use the rigid-rotor approximation

$$
\checkmark \text { Ex: diatomic molecule } \rightarrow \frac{\hbar^{2}}{2 \mu R^{2}} J^{2} \approx \frac{\hbar^{2}}{2 \mu R_{e}^{2}} J^{2} \rightarrow B_{e} J(J+1)
$$

$>\mathrm{HO}+\mathrm{RR}=\omega(v+1 / 2)+B_{e} J(J+1)$
Cannot reproduce the experimental data!!


# Nuclear motion Hamiltonian: different strategies 

Rectilinear (normal) versus curvilinear models?


## Nuclear motion Hamiltonian: different strategies

$>$ The PES is generally expressed in terms of internal (curv.) coordinates (e.g. bond-length, bond-angle)
$>$ For rigid molecules, we have $H=K E O$ (normal coord. $Q$ ) + PES (internal coord. $R$ )
$\Rightarrow$ the way to proceed is to find the nonlinear relation

$$
R=f(Q) \text { in the PES }
$$

$\Rightarrow$ But, we could also try to derive the nonlinear relation

$$
Q=g(R) \quad \text { in the KEO }
$$

> For nonrigid molecules, we have either
or

$$
\begin{aligned}
& H=\operatorname{KEO}(R)+\operatorname{PES}(R) \\
& H=\operatorname{KEO}(Q, \rho)+\operatorname{KEO}(R, \rho) \text { (mixing curv-rect) }
\end{aligned}
$$

The Hamiltonian model: rigid molecules

Watson Hamiltonian (nonlinear molecules, 1968)

nuclear KEO

## PES ab initio

In normal coord.

## The Hamiltonian model: rigid molecules

## Sum of products

## If small amplitude vibrations $\boldsymbol{\rightarrow}$ Taylor series expansion around a single minimum

$$
H=\sum_{i} h_{i}\left(X_{1}^{m_{1, i}} Y_{1}^{n_{1, i}}\right) \cdots\left(X_{3 N-6}^{m_{3 N-6, i}} Y_{3 N-6}^{n_{3 N-6, i}}\right) \cdots
$$

e.g. $X=$ coordinate
$Y=$ conjugate momentum

$$
H^{\left(\Gamma_{0}\right)}=\sum_{i} t_{i}\left(\left(\mathcal{X}_{1} \otimes \mathcal{Y}_{1}\right) \otimes \cdots \otimes\left(\mathcal{X}_{N_{m}} \otimes \mathcal{Y}_{N_{m}}\right)\right)_{i}^{\left(\Gamma_{0}\right)}
$$

Each matrix is now divided in blocks


## The Hamiltonian model: rigid molecules

Symmetry is one the keys to reduce dimensionality of a given problem

In turn, its implementation in computer codes may be rather involved (e.g. non-Abelian points)

The TENSOR computer code (Reims) is able to deal with arbitrary point groups: $C_{n}, C_{n v} D_{n}, D_{n k} D_{n d} T_{d}, O_{h}$

# The Hamiltonian model: <br> nonrigid molecules in Hougen-Bunker-Johns 

If some vibrations are of large amplitude $\boldsymbol{\rightarrow}$ Taylor series expansion fails!

$$
H^{\left(\Gamma_{0}\right)}=\sum_{i} t_{i}\left(\left(\mathcal{X}_{1} \otimes \mathcal{Y}_{1}\right) \otimes \cdots \otimes\left(\mathcal{X}_{N_{m}} \otimes \mathcal{Y}_{N_{m}}\right)\right)_{i}^{\left(\Gamma_{0}\right)} \quad \mathbf{N O}!!
$$


Quantum chemical calculations ..... OK
Nuclear motion Hamiltonian ..... OK
Solving the nuclear problem. ..... ??

## Step 2 - Solving the nuclear Schrödinger equation

> Eigenproblem

$$
H \Psi=E \Psi
$$

> Very few exactly solvable models with known, analytical solutions
$>$ Need to find approximate solutions
$\checkmark$ Approximate the wavefunctions: variational calculations (part I.)
$\checkmark$ Approximate the Hamiltonian: derivation of an effective model (part II.)

# Part. I. Variational calculations 

«The curse of dimensionality »

## Why does $a b$ initio make sense?

Limitations of the traditional spectroscopic, effective approach

Include most of the resonances between rovibrational states

Example 1: For Titan modeling up to $13400 \mathrm{~cm}^{-1}, 3800$ vibrational $\mathrm{CH}_{4}$ sublevels were considered (only 100 from $\mathrm{H}_{\text {eff }}$ )
Example 2: For Exoplanets, more than 10 millions of ro-vibrational $\mathrm{CH}_{4}$ energy levels were computed

Example 3: For $\mathrm{CF}_{4}$ up to $4000 \mathrm{~cm}^{-1}$, about 800 vibrational sublevels were considered (only few ones analyzed from $\mathrm{H}_{\text {eff }}$ )
Example 4: For $\mathrm{SF}_{6}$ up to $3000 \mathrm{~cm}^{-1}$, about 2600 vibrational sublevels, $500 \mathrm{CB} \& \mathrm{HB}$ (only few ones analyzed from $\mathrm{H}_{\text {eff }}$ )

## Solving the vibrational problem



The stationary SE is first solved for $J=0$ by searching the eigenfunctions as


## Primitive functions: example

$$
\Psi_{v, k}(\mathbf{X})=\sum_{i} c_{k}^{i} \phi_{v, i}\left(\mathbf{X} ;\left[p_{1}, p_{2}, \cdots\right]\right)
$$

$\phi_{v, i}(\mathbf{X})=$ Harmonic oscillator function are KNOWN
... not "physical" functions BUT a combination of many HO functions
can describe properly the TRUE, UNKNOWN wavefunctions
$=$ the primitive functions have to be conveniently chosen to converge rapidly calc.

## Matrix elements

The matrix elements of H in the primitive basis

## The matrix elements of in the primitive basis


can be evaluated $\int_{\text {row }}^{\phi_{v^{\prime}}^{*}(\mathbf{X}) H}{\underset{v}{\text { column }}}_{(\mathbf{X})} d \mathbf{X}$
$>$ either analytically / algebraically $\rightarrow$ exact \& fast but not flexible

v
\(\xrightarrow{\substack{c <br>
\phi_{1} <br>

\phi_{2}}}\)|  | $\cdots$ | $\cdots$ | $\phi_{m}$ |
| :--- | :--- | :--- | :--- |


$\mathbf{v}^{\prime} |$| $\phi_{1}$ |
| :---: |
| $\phi_{2}$ |
| $\vdots$ |
| $\vdots$ |
| $\phi_{m}$ |


Diagonalization = Energy levels
$>$ or numerically (quadrature) $\rightarrow$ much more flexible

## Direct product primitive basis set

The simplest way to build a primitive vibrational basis consists in forming a direct product of $3 N-6$ functions

$$
\phi_{v}(\mathbf{X})=\phi_{v_{1}}\left(X_{1}\right) \phi_{v_{2}}\left(X_{2}\right) \cdots \phi_{v_{3 N-6}}\left(X_{3 N-6}\right) \equiv\left|v_{1} v_{2} \cdots v_{3 N-6}\right\rangle
$$

Note: non direct product basis sets are also possible

Imagine 10 functions per degree of freedom
$\checkmark$ Triatomic molecules: number of basis functions $=10^{3}=>0.008 \mathrm{~Gb}$ to store the H matrix
$\checkmark$ Tetraatomic molecules: number of basis functions $=10^{6}=>8 \mathrm{~Tb}$ to store the H matrix in memory !
$\checkmark$ Pentaatomic molecules: number of basis functions $=10^{9}=>8000000 \mathrm{~Tb}$ to store the H matrix !!!

## Direct product primitive basis set

Typically, each time we add an atom, the Hamiltonian matrix is scaled by a factor 1000 without approximation!


Need to find compromises

A drastic dimensionality reduction is required

Several ways to proceed

## Pruning the basis

To drastically reduce dimensionality by several orders of magnitude, a "pruned basis" is introduced and defined by selecting only a limited set of relevant primitive vibrational functions
with $v_{i}=0, \cdots, r$


The dimension of the basis is reduced from $\sim r^{(3 N-6)}$ to $\sim(3 N-6+r)!/(3 N-6)!r!$

Illustration : for a 5-atomic molecule, the number of basis functions is reduced from $10^{9}$ to $10^{5}$

## Use of symmetry-adapted functions

The use of symmetry can reduce the dimension of the H matrix up to 1 order of magnitude

A course on group theory is required here...

Pruning and symmetry may not be sufficient

## Contracted basis functions

We can consider reduced Hamiltonians and solve SE of lower dimension before gathering all eigenfunctions
Imagine a full 6D Hamiltonian

$$
H\left(X_{1}, Y_{1} ; X_{2}, Y_{2} ; X_{3}, Y_{3} ; X_{4}, Y_{4} ; X_{5}, Y_{5} ; X_{6}, Y_{6}\right)
$$

Let 2 subgroups as

$$
\begin{aligned}
& G_{1}=\left\{X_{1}, Y_{1} ; X_{2}, Y_{2} ; X_{3}, Y_{3} ; X_{4}, Y_{4}\right\} \\
& G_{2}=\left\{X_{5}, Y_{5} ; X_{6}, Y_{6}\right\}
\end{aligned}
$$

such that

$$
H=H_{G_{1}}+H_{G_{2}}+H_{G_{1}-G_{2}}
$$

## Contracted basis functions

We solve separately the SE

$$
\begin{aligned}
& H_{G_{i}} \Psi_{G_{i}}=E_{G_{i}} \Psi_{G_{i}} \\
& \Psi_{G_{i}}=\mathrm{U}_{G_{i}}^{-1} \Phi_{G_{i}} \quad \text { with } \quad \mathrm{U}_{G_{i}} \in \mathbb{R}^{m_{i} \times m_{i}}
\end{aligned}
$$

In practice, we retain only $n_{i} \ll m_{i}$ eigenvectors to reduce dimensionality without almost no loss of precision

$$
\begin{array}{lllll}
\Psi_{1 G} & \Psi_{2 G} & \cdots & \cdots & \Psi_{m G}
\end{array}
$$



## Contracted basis functions

$>$ In the primitive basis $\Phi_{G_{1}} \times \Phi_{G_{2}}$ the Hamiltonian matrix is huge, making calculations not tractable
$>$ In the eigenvector basis $\Psi_{G_{1}} \times \Psi_{G_{2}}$ the matrix representation of $H=H_{G_{1}}+H_{G_{2}}+H_{G_{1}-G_{2}}$ reads


Ex: $H_{G 1}=$ Stretch, $H_{G 2}=$ Bend
$H_{G 1}=$ molecule $1, H_{G 2}=$ molecule $2 \Rightarrow H=$ van der Waals complex

## Reducing the memory cost of the matrix multiplication

> Principal Component Analysis (PCA), SVD
$>$ Low-rank tensor approximations (canonical polyadic, Tucker format, tensor train decomposition) (MCTDH-type calculations)


Reducing the memory cost / compromise with accuracy


## Choice of the eigensolver

If at the end of the procedure ....
$>\operatorname{Dim}($ Matrix $)<100000 \times 100000 \rightarrow$ direct eigensolvers (Jacobi, householder, etc): LAPACK library Fit in memory
$>\operatorname{Dim}($ Matrix $)>100000 \times 100000$
$\rightarrow$ iterative methods (Lanczos/Arnoldi) will be preferred Does not fit in memory

## Lanczos: basic principles

$>$ Avoids diagonalization of huge matrices - Not demanding
> The basic idea is to transform the HUGE H matrix into a tri-diagonal matrix

$>$ T is built iteratively (Lanczos iterations) - The exact arithmetic algorithm is very easy to implement
> The eigenvalues converge rapidly (from the bottom and the top) if the spectrum is not too dense
$>\ldots$. But to be used with care .... the orthogonality of the Lanczos vectors may be lost due to rounding errors

## Vibrational part solved

## Next step: introduction of the molecular rotation

## 3 additional degrees of freedom

Not good quantum numbers


## Introduction of the molecular rotation

$>$ For high $J$ calculations, I have introduced so-called reduced vibrational eigenfunctions of lower dimension

$$
\begin{array}{cc}
\Psi_{v, i}^{\left(C_{v}\right)}=\sum_{j \in \mathcal{F}_{p}} U_{j, i}^{\left(C_{v}\right)} \phi_{v, j}^{\left(C_{v}\right)} & \mathbf{U} \in \mathbb{R}^{M \times M} \\
\Psi_{v, i}^{\left(C_{v}\right)}=\sum_{j \in \mathcal{F}_{p^{\prime}}} U_{j, i}^{\left(C_{v}\right)} \phi_{v, j}^{\left(C_{v}\right)}+\sum_{k \notin \mathcal{F}_{p^{\prime}}} U_{k, i}^{\left(C_{v}\right)} \phi_{v, k}^{\left(C_{v}\right)} & \begin{array}{l}
\text { Small in } \\
\text { the range of OBS }
\end{array} \\
\Psi_{\text {app }, v, i}^{\left(C_{v}\right)}=\sum_{j \in \mathcal{F}_{p^{\prime}}} U_{\text {app,j,i}}^{\left(C_{v}\right)} \phi_{v, j}^{\left(C_{v}\right)} & \mathbf{U}_{a p p} \in \mathbb{R}^{M^{\prime} \times M^{\prime}} \\
\left(M^{\prime} \ll M\right)
\end{array}
$$

## Illustration on the first full-dimensional quantum mechanical calculations for a 7-atomic molecule: Case of $\mathrm{SF}_{6}$



## Key results

- 2600 calc. band centers up to $3000 \mathrm{~cm}^{-1}$
- 3 millions of levels up to $J=120$
- 6 billion lines in RT spectra up to $3000 \mathrm{~cm}^{-1}$


# Quality of the calculation: Precision versus Accuracy 

$>2$ ways that scientists think about error
$\checkmark$ Precision is how close different calculations are to each other
$\checkmark$ Accuracy is how close a calculation is to experiment

We have to control the precision of calculation to reach high accuracy


Non converged energy levels or errors during calculations (loss of precision, bug, etc)
precise, not accurate



Good calculation but « bad » PES

precise, accurate

Good calculation
using a refined or accurate PES

## Accuracy of $a b$ initio calculations



Finally, why such theoretical developments?
$>$ Example 1 : calculation of the molecular line list for methanol \& ethane


Many degrees of freedom + large amplitude $\rightarrow$ great challenge


## Finally, why such theoretical developments?

$>$ Example 2: studying small PAH (Polycyclic Aromatic Hydrocarbons) in a near future, why not?


## At this stage

(1) PES \& DMS OK (commercial codes)
(2) Solving the Schrödinger equation OK (home-made codes)
$(1)+(2)=$ theoretical line lists

DMS (quantum chemists) + variational eigenfcts

## Converging integrated intensity for consistent opacity calculation



Convergence is governed by 3 parameters

1. J angular rot. Momentum
2. $\mathrm{E}_{\text {low }}$ value for including HB
3. $\mathrm{I}_{\text {cutoff }}$ value

WARNING
Need to be converged simultaneously !!

## Example: methane spectra

Titan


## Managing the huge number of lines

Q: How to manage billions of lines?
R: data compression using "superlines": from several billions to few millions !! The user is now HAPPY ©


## Validation @3 $\mu$

## $\mathrm{CH}_{4}$ EXP : R. Georges et al. (Rennes) 1480K




## Validation @1.3 \& $1 \mu$



Validation @ high T (500 and 1000 K )



## Major methane publications

## http://theorets.univ-reims.fr



# Other molecules 

## Phosphine

Same strategy as $\mathrm{CH}_{4} \rightarrow$ empirically corrected line lists (for positions) based on Andrei Nikitin's analysis Best of both worlds $\rightarrow$ accurate line positions for cold \& hot bands + accurate $a b$ initio intensities





All vibrational bands are systematically taken into account





## Isotopic species

- Predicting all isotopologues almost simultaneously




## Ammonia (nonrigid)



## Variational calculation: summary

$\checkmark$ Completeness OK
$\checkmark$ Consistent opacity calculation because all resonances are taken into account
$\checkmark$ Relevant for planetary applications
BUT .....
$\checkmark$ Very involved calculations - it is difficult to refine the PES
$\checkmark$ Not able to reach the HR spectroscopic accuracy ( $\sim 0.001 \mathrm{~cm}^{-1}$ in the infrared)

## Part. II. Effective models

$\checkmark$ Small dimensionality: block-diagonal transformation of the Hamiltonian
$\checkmark$ Can reach spectroscopic accuracy - only a small spectral range can be studied
$\checkmark$ Assist a spectroscopist in analysis


## Basic principles of the (empirical) effective approach

Transform a full problem into a series of much smaller problem = POLYADS

$\Rightarrow$ Describes a small group of interacting vibrational states $=$ 1 effective Hamiltonian per group

$$
\text { Line position }=E_{j}(\tilde{t})-E_{i}(\tilde{t})
$$

Least squares fitting to determine the parameters - data reduction

Empirical effective Hamiltonians: ideal case


Empirical effective Hamiltonians: ideal case

Block-diagonal Hamiltonian matrix


0


Empirical effective Hamiltonians: practical case


Empirical effective Hamiltonians: practical case


## Empirical effective Hamiltonians


$>$ Very efficient for isolated polyads (small dimensionality=accurate models) (MW spectroscopy, few vibrational bands)
$>$ Turn out very limited for complex polyads involving many interacting vibrational states

Effective Hamiltonian: OK

## Alternative: variational approach (based on ab initio PES)



Global calculation - much more consistent
All the resonance coupling terms are taken into account
Very good representation of the wavefunctions for line intensities


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$$
\Psi_{v, k}(\mathbf{X})=\sum_{i} c_{k}^{i} \phi_{v, i}(\mathbf{X})
$$



HUGE matrix to be diagonalized $\Theta^{\circ}$

Energy levels of methane: effective vs. variational


Titan


Energy levels of methane: effective vs. variational


## Combining the best of both worlds ? ab initio effective approach

Transform a full problem into a series of much smaller problem


Determine from both the PES \& geometry
$\checkmark$ Small dimensionality
$\checkmark$ Almost as complete as variational calculations

## $a b$ initio effective model (Van Vleck algebraic procedure)

Transformation of the nuclear Hamiltonian

Usual perturbative procedure $\exp \left(-i \lambda S_{1}\right) * \exp \left(-i \lambda^{2} S_{2}\right) * \exp \left(-i \lambda^{3} S_{3}\right) \ldots \quad \rightarrow$


Generators to be found

$$
\begin{aligned}
\boldsymbol{H}_{0}^{(1)}= & \boldsymbol{H}_{0}, \\
\boldsymbol{H}_{1}^{(1)}= & \boldsymbol{H}_{1}+i\left[\mathbf{S}_{1}, \boldsymbol{H}_{0}\right], \\
\boldsymbol{H}_{2}^{(1)}= & \boldsymbol{H}_{2}+i\left[\mathbf{S}_{1}, \boldsymbol{H}_{1}\right]-\frac{1}{2}\left[\mathbf{S}_{1},\left[\mathbf{S}_{1}, \boldsymbol{H}_{0}\right]\right], \\
\boldsymbol{H}_{3}^{(1)}= & \boldsymbol{H}_{3}+i\left[\mathbf{S}_{1}, \boldsymbol{H}_{2}\right]-\frac{1}{2}\left[\mathbf{S}_{1},\left[\mathbf{S}_{1}, \boldsymbol{H}_{1}\right]\right]-\frac{1}{6} i\left[\mathbf{S}_{1},\left[\mathbf{S}_{1},\left[\mathbf{S}_{1}, \boldsymbol{H}_{0}\right]\right]\right] \\
\boldsymbol{H}_{4}^{(1)}= & \boldsymbol{H}_{4}+i\left[\mathbf{S}_{1}, \boldsymbol{H}_{3}\right]-\frac{1}{2}\left[\mathbf{S}_{1},\left[\mathbf{S}_{1}, \boldsymbol{H}_{2}\right]\right]-\frac{1}{6} i\left[\mathbf{S}_{1},\left[\mathbf{S}_{1},\left[\mathbf{S}_{1}, \boldsymbol{H}_{1}\right]\right]\right] \\
& +\frac{1}{24}\left[\mathbf{S}_{1},\left[\mathbf{S}_{1},\left[\mathbf{S}_{1},\left[\mathbf{S}_{1}, \boldsymbol{H}_{0}\right]\right]\right]\right]
\end{aligned}
$$

Millions of commutators for polyatomic molecules Transformed dipole moment - tricky task

$$
H_{n}^{(1)}=M_{n}+\sum_{m=0}^{n-1}\left(i^{n-m} /(n-m)!\right) \underbrace{\left[S_{1},\left[S_{1}, \ldots,\left[S_{1}, 1,\right.\right.\right.}_{n-m}], \ldots]]
$$

Matrix transformation

Without using perturbation theory - Obviate the need to perform very involved CT
$\checkmark$ Numerical approach for the block diagonalization (no need to compute commutators)


## Numerical procedure?



Basic principles : numerical transformation of selected variational eigenpairs

$$
\left\{E, \boldsymbol{\Psi}=\mathbf{U}^{-1} \boldsymbol{\Phi}\right\}
$$

Huge matrix


Very small blocks for each polyad and each symmetry block

Complete nuclear-motion Hamiltonian (input)
Hamiltonian matrix, Eq. (6)
$\Omega$
Eigenpair, Eq. (8)


Effective Hamiltonian, Eq. (17)
Effective Dipole, Eq. (20)
calculated spectrum (output)

## $a b$ initio effective Hamiltonian


transformation


Illustration: $\mathrm{PH}_{3}$ parameters for the dyad

| Operator | $\bar{s}$, Eq. (19) |  | $\bar{s}, \mathrm{fit}^{a}$ |
| :---: | :---: | :---: | :---: |
| Ground state ( $P=0$ ) | Fro | m PES, this work | From FIT |
| $R^{2(2,041)}$ |  | $-1.08859121 \times 10^{-1}$ | $-1.08856(2) \times 10^{-1}$ |
| $R^{2(0,0 A 1)}$ |  | -1.85097033 | -1.850969(1) |
| $R^{4(4,0 A 1)}$ | $\bigcirc$ | $-4.21792347 \times 10^{-6}$ | $-4.224(3) \times 10^{-6}$ |
| $R^{4(4,3 A 1)}$ | " | $6.35064638 \times 10^{-6}$ | $6.393(9) \times 10^{-6}$ |
| $R^{4(2,0 A 1)}$ | $\begin{array}{ll} \bar{O} & \mathbb{U} \\ \subset \end{array}$ | $-4.54280135 \times 10^{-6}$ | $-4.530(5) \times 10^{-6}$ |
| $R^{4(0,0 A 1)}$ |  | $-1.91476610 \times 10^{-5}$ | $-1.9143(3) \times 10^{-5}$ |
|  |  | $\mathrm{RMS}=0.0018$ | RMS $=0.00013^{\text {b }}$ |
| $\operatorname{Dyad}(P=1)$ |  |  |  |
| $R^{0(0,0 A 1)}\left(a_{2}^{+} \otimes a_{2}\right)^{\left(A_{1}\right)}$ |  | 992.134856 | 992.1354(1) |
| $R^{2(2,0 A 1)}\left(a_{2}^{+} \otimes a_{2}\right)^{\left(A_{1}\right)}$ |  | $6.11040167 \times 10^{-3}$ | $5.877(5) \times 10^{-3}$ |
| $R^{2(0,0 A 1)}\left(a_{2}^{+} \otimes a_{2}\right)^{\left(A_{1}\right)}$ |  | $-2.85659105 \times 10^{-3}$ | $-2.863(6) \times 10^{-3}$ |
| $R^{0(0,0 A 1)}\left(a_{4}^{+} \otimes a_{4}\right)^{\left(A_{1}\right)}$ |  | 1581.52444 | 1581.5238(1) |
| $R^{2(2,0 A 1)}\left(a_{4}^{+} \otimes a_{4}\right)^{\left(A_{1}\right)}$ | $0 \text { 亿 }$ | $-1.05612963 \times 10^{-2}$ | $-1.0614(5) \times 10^{-2}$ |
| $R^{2(0,0 A 1)}\left(a_{4}^{+} \otimes a_{4}\right)^{\left(A_{1}\right)}$ | $\bigcirc$ ¢ | $-1.12134942 \times 10^{-3}$ | $-1.119(6) \times 10^{-3}$ |
| $R^{2(2,1 E)}\left(a_{4}^{+} \otimes a_{4}\right)^{(E)}$ |  | $2.18084810 \times 10^{-2}$ | $2.176(3) \times 10^{-2}$ |
| $R^{2(2,2 E)}\left(a_{4}^{+} \otimes a_{4}\right)^{(E)}$ |  | $-5.13139342 \times 10^{-5}$ | $-3.52(7) \times 10^{-5}$ |
|  |  | RMS $=0.019$ | RMS $=0.00057^{\text {b }}$ |




## Hamiltonian parameters for $\mathbf{H}_{\mathbf{2}} \mathbf{C O}$

Dipole moment parameters for $\mathbf{H}_{2} \mathbf{C O}$


40 years


Extension up to $20,000 \mathrm{~cm}^{-1}$ in progress



## Ex: @ 3 microns

- 4 bands in HITRAN (not complete)
- 52 bands in this work


Towards a change of paradigm in high-resolution spectroscopy?

- Current models: 10-20 bands
- This model: >500 bands with corresponding parameters in few hours


I hope you are now convinced that ab initio is useful for the modelling of planetary atmospheres and that the construction
of line list is far from being simple...

## Many molecular lists can be now calculated

```
\checkmark CH
\ CH3D, CH3Cl, CH3F, CH3
\checkmark CH
\checkmark SiH
\checkmark C3 H4},\mp@subsup{S}{4}{}\mp@subsup{\textrm{SiF}}{4}{},\mathrm{ etc.
\checkmark C}\mp@subsup{\textrm{C}}{6}{}\mp@subsup{\textrm{H}}{6}{}\mathrm{ ? C C4 H4}\mp@subsup{\textrm{H}}{4}{}\mp@subsup{\textrm{O}}{2}{}\mp@subsup{\textrm{N}}{2}{}\mathrm{ (uracil) ? }\mp@subsup{\textrm{C}}{8}{}\mp@subsup{\textrm{H}}{10}{}\mathrm{ (naphtalene)?
\checkmark ~ \mathrm { C } _ { 2 } \mathrm { H } , \mathrm { C } _ { 2 } \mathrm { H } _ { 2 } \text { (PES OK, codes to be adapted to linear molecules)}
\ C2 H4 O (no DMS)
\checkmark + all isotopologues
```

Please suggest me other molecules

