Atmospheric models for retrieval and interpretation

Benjamin Charnay

Ecole des Houches, 2022
20 years of exoplanet atmospheric characterization

- Exoplanet atmospheric observations are now done routinely
- Observations revealed a great diversity of atmospheres, many of them seem cloudy/hazy
- Plenty molecules/atoms detected \((\text{H}_2\text{O}, \text{CO}, \text{CH}_4, \text{NH}_3, \text{HCN}, \text{CO}_2, \text{C}_2\text{H}_2, \text{H}, \text{He}, \text{Na}, \text{K}, \text{Cr}, \text{V}, \text{Fe}, \text{FeH}, \text{TiO}, \text{VO}, \text{C}^{13}\text{O})\)

Guillot et al. 2022
Main goal = understand the diversity of exoplanets (from Ariel Red book) :

1) Which physical/chemical processes shape exoplanet atmospheres?
2) What are the compositions/physical conditions of exoplanet interiors?
3) Which processes control exoplanet formation and evolution?
What is the thermal structure of exoplanets and how is it shaped?

➢ Which planets have a stratospheric thermal inversion?

➢ What is the impact of clouds, atmospheric dynamics or fingering convection?
Atmospheres as a probe of planetary interior and formation

Metallicity = fraction of heavy elements (heavier than H and He)
For Solar System atmospheres, metallicity ≈ [C]/[C]_{solar}
For exoplanetary atmospheres, metallicity ≈ [O]/[O]_{solar}

- Metallicity decreases with planetary mass in the Solar System
- Short-period planets formed in-situ should have a relatively low metallicity

→ Metallicity measurements give constraints on formation and migration mechanisms
Atmospheric C/O may depend on where the planet formed
High C/O => gas accretion
Low C/O => enrichment by planetesimal

C/O may decrease for low-mass planets

The reality is certainly much more complex

→ C/O measurements give constraints on formation mechanisms
Yes, it is definitively flat! 10b$ for that...

No atmosphere, clouds or high mean molecular mass

e.g. TauREX

Is it really flat?

Atmospheric parameters

Forward model + Retrieval techniques

Data

Interpretation
1D Forward models

Radiative transfer: transit spectroscopy

Optical depth (cross-section):

$$\tau(r, \lambda) = \sum_i \int_{-\infty}^{+\infty} \sigma_i(\lambda, P, T)n_i(z)dx$$

Transit depth:

$$D(\lambda) = \left(\frac{R_p}{R_*}\right)^2 + 2 \left(\frac{R_p}{R_*}\right)^2 \int_{R_p}^{R_*} r(1 - e^{-\tau(r, \lambda)})dr = \left(\frac{R_p + h_\lambda}{R_*}\right)^2$$

Equivalent altitude:

$$h_\lambda \approx r(\tau = 0.56) - R_p$$

see De Wit & Seager (2013) and Macdonald & Cowan (2019)
1D Forward models

Radiative transfer: transit spectroscopy

Synthetic Earth’s transit spectrum

Macdonald & Cowan (2019)
Real color

Credit: Himawary/Simon Proud/Vivien Parmentier
8.6 microns spectral window (15 km)

Credit: Himawary/Simon Proud/Vivien Parmentier
9.6 microns
O$_3$ band (40 km)

Credit: Himawary/Simon Proud/Vivien Parmentier
1D Forward models

Radiative transfer: transit spectroscopy

**Ideal case:**
- hydrostatic+isothermal
- cross-sections independent of P & T
- constant abundances

$$\tau(b, \lambda) = \sum_i \int_{-\infty}^{+\infty} \sigma_i(\lambda)n_i(x)dx \ ; \ p(z) = p(z_0) \exp\left(-\frac{z-z_0}{H}\right) \text{ with } H = \frac{RT}{Mg}$$

$$n_i(x) = n_i0 e^{-z/H} \text{ with } z = \sqrt{r^2 + x^2} - Rp \approx r - Rp + \frac{x^2}{2Rp}$$

$$\tau(r, \lambda) \approx \sum_i \sigma_i(\lambda)n_i0 e^{-(r-Rp)/H} \int_{-\infty}^{+\infty} e^{-x^2/2RpH} dx = \sum_i \sigma_i(\lambda)n_i0 e^{-(r-Rp)/H} H \sqrt{2\pi Rp \frac{H}{H}}$$

Comparison with vertical optical depth:

$$\eta = \frac{\tau_H}{\tau_v} = \sqrt{\frac{2\pi Rp}{H}}$$

Earth: $\eta \sim 75$
K2-18 b: $\eta \sim 60$
HD209458 b: $\eta \sim 38$

Transits probe pressures 1-2 orders of magnitude lower than eclipses
# 1D Forward models

## Radiative transfer: transit spectroscopy

**Ideal case:**
- hydrostatic+isothermal
- cross-sections independent of P & T
- constant abundances

\[
\tau(b, \lambda) = \sum_i \int_{-\infty}^{+\infty} \sigma_i(\lambda)n_i(x)dx \quad ; \quad p(z) = p(z_0) \exp\left(-\frac{z-z_0}{H}\right) \text{ with } H = \frac{RT}{Mg}
\]

\[
n_i(x) = n_i0e^{-z/H} \text{ with } z = \sqrt{r^2 + x^2 - Rp} \approx r - Rp + \frac{x^2}{2Rp}
\]

\[
\tau(r, \lambda) \approx \sum_i \sigma_i(\lambda)n_i0e^{-(r-Rp)/H} \int_{-\infty}^{+\infty} e^{-x^2/2RpH}dx = \sum_i \sigma_i(\lambda)n_i0e^{-(r-Rp)/H} H \sqrt{\frac{2\pi Rp}{H}}
\]

**Variation of transit depth:**
\[
\tau_1(r_1, \lambda_1) = \tau_2(r_2, \lambda_2) \approx 0.56 \quad ; \quad \Delta r = H\ln\left(\frac{\sigma(\lambda_1)}{\sigma(\lambda_2)}\right) ; \quad \Delta D = \frac{2RpH\ln\left(\frac{\sigma(\lambda_1)}{\sigma(\lambda_2)}\right)}{R_s^2}
\]

- Transit spectroscopy easier for high scale height (e.g. hot giant planets)
  \[
  \frac{\sigma(\lambda_{\text{max}})}{\sigma(\lambda_{\text{min}})} \approx 10^3 \rightarrow \Delta r \approx 7H
  \]
- Transit depth at low resolution depends on the mean value of \(\ln(\sigma)\)
1D Forward models

Radiative transfer: transit spectroscopy

Opacity of H₂O and CH₄ (at 300K & 1 mbar) computed with the online tool DACE/OPACITY (https://dace.unige.ch/opacity/)

\[
\frac{\sigma_{H_2O}}{\sigma_{CH_4}} \approx 10; \quad \frac{\ln(\sigma_{H_2O})}{\ln(\sigma_{CH_4})} \approx 0.1
\]

H₂O should be masked by CH₄ at low resolution but not at high resolution.
1D Forward models

Radiative transfer: transit spectroscopy

Example K2-18 b:

K2-18b:
Mass = 8.63 M⊕
Radius = 2.6 R⊕
Irradiation = 1368 W/m²
(1361 W/m² for the Earth)
Orbital period = 33 days

A temperate sub-Neptune, with water vapour and potentially water clouds

HST transit spectrum

Tsiaras et al. (2019)

Benneke et al. (2019)
1D Forward models

Radiative transfer: transit spectroscopy

Example K2-18 b:

Transit spectra of K2-18 b computed with Exo-REM (metallicity=200×solar)

Bézard, Charnay & Blain (2022)

CH₄ should be the dominant absorber for a solar C/O
H₂O should be the dominant absorber for C/O<0.1×C/O₅olar
1D Forward models

Radiative transfer: definition of intensity and flux

**Intensity** $I$ = amount of energy passing through a surface area $dS$, within a solid angle $d\Omega$, per wavelength interval $d\lambda$, per unit time ($I$ in J m$^{-2}$ sr$^{-1}$ μm$^{-1}$):

$$dE = I(x, \vec{n}, \lambda, t) \vec{n} \cdot \vec{k} \, d\Omega \, dS \, d\lambda \, dt$$

**Moments:**

Mean intensity:

$$J = \int_{\Omega} I(x, \vec{n}, \lambda, t) \, d\Omega$$

Flux:

$$F = \int_{\Omega} I(x, \vec{n}, \lambda, t) \vec{n} \cdot \vec{k} \, d\Omega = \int \int I(x, \theta, \phi, \lambda, t) \cos(\theta) \sin(\theta) \, d\theta \, d\phi$$
1D Forward models

Radiative transfer: equation for plane-parallel

Optical depth & extinction coefficient:
\[
d\tau = -k(T, P, \lambda) \mu \, ds
\]
\[
k(T, P, \lambda) = \sum \ n_i (\sigma_i^{abs} + \sigma_i^{scat})
\]
Optical mean free path: \( l = \frac{1}{k} \)

Radiative transfer equation:
\[
\mu \ \frac{dl}{d\tau} = I - S
\]

Local thermodynamic Equilibrium (LTE):
\[
T_{\text{radiation}} = T_{\text{kinetics}}
\]
mean free path of photons \( \ll \) length scale of \( T \) variations
1D Forward models

Solution a purely emitting atmosphere

\[ \mu \frac{dI}{d\tau} = I - B \]

\[ B(T, \lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \]

Intensity at top of the atmosphere:

\[ I(\tau = 0, \mu, \phi, \lambda) = B(T(\tau_0), \lambda) e^{-\tau_0/\mu} + \int_0^{\tau_0} B(T(\tau), \lambda) e^{-\tau/\mu} \frac{d\tau}{\mu} \]

Flux at top of the atmosphere (outgoing radiation):

\[ F^\uparrow(\tau = 0, \lambda) = \int_0^{2\pi} \int_0^1 \mu I(\tau = 0, \mu, \phi, \lambda) d\mu d\phi \]

Resolution with Gauss-quadrature:

\[ F^\uparrow(\tau = 0, \lambda) = 2\pi \sum_{i}^{N} \mu_i I(\tau = 0, \mu_i, \lambda) \omega_i \]
The two-stream approximation

\[ \mu \frac{dI}{d\tau} = I - S \]

**Goal:** to compute the total upward and downward flux

\[
\begin{align*}
J^\uparrow &\equiv \int_0^{2\pi} \int_0^1 I \, d\mu \, d\phi, \\
J^\downarrow &\equiv \int_0^{2\pi} \int_{-1}^0 I \, d\mu \, d\phi, \\
F^\uparrow &\equiv \int_0^{2\pi} \int_0^1 \mu I \, d\mu \, d\phi, \\
F^\downarrow &\equiv \int_0^{2\pi} \int_{-1}^0 \mu I \, d\mu \, d\phi,
\end{align*}
\]

The two-stream solution consists in approximating \( I \) so that it is related to \( F \).

We assume \( \frac{F^\uparrow}{J^\uparrow} = \frac{F^\downarrow}{J^\downarrow} = \frac{1}{\gamma} \iff \mu = \frac{1}{\gamma} \)

(generally \( \gamma = \sqrt{3} \))
1D Forward models
The two-stream approximation

Case of a purely emitting atmosphere:

Outgoing radiation:

\[ F^\uparrow (\tau = 0) = F^\uparrow (\tau_0) e^{-\gamma \tau_0} + \int_0^{\tau_0} 2\pi \gamma B e^{-\gamma \tau} d\tau \]

Transmittance

Weighting function:

\[ cf(P) = B(\lambda, T) \frac{de^{-\gamma \tau}}{d\log(P)} \]

Peak of contribution:
at \( \tau \sim 2/3 \) also called the photosphere

Brightness temperature:

\[ T_b \sim T(\tau = 2/3) \]
1D Forward models
Link between thermal structure and emission

Case of a purely emitting atmosphere:

Variation of thermal flux:

\[
\frac{\delta F}{F} \approx - \frac{T}{B} \frac{\partial B}{\partial T} \frac{d\ln T}{k} \delta k
\]

The emission spectrum with/without thermal inversion

For \( \delta k > 0 \):

• \( \frac{\delta F}{F} < 0 \rightarrow \frac{dT}{dz} < 0 \) (no thermal inversion)
• \( \frac{\delta F}{F} > 0 \rightarrow \frac{dT}{dz} > 0 \) (thermal inversion)
• \( \frac{\delta F}{F} \approx 0 \rightarrow \frac{dT}{dz} \approx 0 \) (isothermal)
1D Forward models

Link between thermal structure and emission

Where is the signature of the stratospheric thermal inversion in the emission spectrum?
1D Forward models

Link between thermal structure and emission

Earth’s thermal emission
1D Forward models

Methods for solving RT

General case of the two-stream approximation (thermal emission + scattering)

\[
\frac{\partial F^\dagger}{\partial \tau} = \gamma_1 F^\dagger - \gamma_2 F^\downarrow - 2\pi(1 - \omega_0)B \\
\frac{\partial F^\downarrow}{\partial \tau} = \gamma_2 F^\dagger - \gamma_1 F^\downarrow + 2\pi(1 - \omega_0)B
\]

<table>
<thead>
<tr>
<th>Method</th>
<th>(\gamma_1)</th>
<th>(\gamma_2)</th>
<th>(\mu_\ast)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrature</td>
<td>(\sqrt{3}[1 - \omega_0(1 + g)/2])</td>
<td>(\sqrt{3}\omega_0(1 + g)/2)</td>
<td>(1/\sqrt{3})</td>
</tr>
<tr>
<td>Hemispheric mean</td>
<td>(2 - \omega_0(1 + g))</td>
<td>(\omega_0(1 - g))</td>
<td>(1/2)</td>
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</tbody>
</table>

Quadrature for deep atmosphere & Hemispheric mean for the upper atmosphere

See Toon et al. (1989) for the complete solution with multi-layers
1D Forward models

Methods for solving RT

1) **Semi-grey analytical model**

> Only for computing the thermal structure (e.g. for retrieval or thermal evolution)

- **Model of Guillot et al. (2010):**
  Two parameters \(k_{vis}\) and \(k_{ir}\) for visible (stellar) and infrared (planetary) radiation

- **Models with sub-bands:**
  - e.g. Parmentier et al. (2014) and Robinson & Catling (2012):
    One parameter for visible \(k_{vis}\) and three parameters for infrared \(k_{ir1}, k_{ir2}, \beta = \frac{\delta v_2}{\Delta v}\)

2) **Correlated-k method**

> Multiple sub-bands representative of the distribution of opacity inside a large band

- **Fast method, excellent for low and medium resolution**
- **Can combine different molecular species**
- **Widely used for atmospheric models and 3D GCM**
1D Forward models

Methods for solving RT

1) Semi-grey analytical model

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2) Correlated-k method

**Exo_k:** tool to compute kcoefficients for different formats
(PCM GCM, Exomol, Nemesis, PetitCode, TauREx, Exo-REM, ARCIS):
http://perso.astrophy.u-bordeaux.fr/~jleconte/exo_k-doc/index.html
1D Forward models

Self-consistent models

Input parameters:
- Planetary radius
- Gravity
- Irradiation
- $T_{\text{int}}$
- Metallicity
- C/H, O/H, N/H,...
- Initial TP profile
- Opacities

Thermal structure

Chemical composition

Cloud distribution

Radiative transfer

Transmission & emission spectra

Iterations until converged (difficult sometimes...)

Models available online:
Exo-REM, PICASO, ATMO, PetitCode, Exo_k

Avantage/disavantages:
+ Physical solutions
- Solutions biased by model parametrization
- Slow (cannot be run online in retrieval)

→ Ideal for limited dataset or limited parameter exploration and for predicting/interpreting observations
1D Forward models

Parametric models

Input parameters:
- Planetary radius
- Gravity
- Abundance
- TP profile
- Cloud profile
- Opacities

Thermal structure
Chemical composition
Cloud distribution

Radiative transfer

Transmission & emission spectra

Avantage/disavantages:
+ Solutions not biased by model parametrization
+ Fast (can be run online in retrieval)
- Can provide unphysical solutions

→ Ideal for atmospheric retrieval without be biased by model parametrizations
Retrieval techniques

Chi2 with model grids

- Simplest method, used in particular for self-consistent models

- Principe:
  - $N$ measurements $F_i^{\text{obs}} \pm \sigma_i$ (uncorrelated)
  - $F_i^{\text{model}}$ from a model
  - Minimization of the cost function:

$$\chi^2 = \sum_{i=1}^{n} \frac{(F_i^{\text{obs}} - F_i^{\text{model}})^2}{\sigma_i^2}$$

- Contours of constant $\chi^2$:

$$\chi^2 = \chi^2_{\text{min}} + \Delta\chi^2$$

Simulation of GJ 504 b with MIRI-MRS (Mâlin et al. in prep):
Retrieval techniques

Chi2 with model grids

JWST-NIRSpec spectrum of WASP-39 b (ERS)
Retrieval techniques

Bayesian inference (MCMC & Nested-sampling)

A simple retrieval with a Guillot TP profile (5 parameters) + 3 molecule abundances (e.g. H$_2$O, CH$_4$, CO) + clouds ($P_{\text{top}}$) + Rp = 10 free parameters!

→ A statistical method is required to explore the parameter space, focusing on the best fits

- Bayesian inference

\[
P(\text{model}|\text{data}) = \frac{P(\text{data}|\text{model})P(\text{model})}{P(\text{data})}
\]

- Posterior probability of the model
- Likelihood function of the data
- Prior probability of the model

Evidence [not important because absorbed into the normalisation of the posterior]
Retrieval techniques

Bayesian inference (MCMC & Nested-sampling)

A simple retrieval with a Guillot TP profile (5 parameters) + 3 molecule abundances (e.g. H$_2$O, CH$_4$, CO) + clouds ($P_{\text{top}}$) + Rp = 10 free parameters!

→ A statistical method is required to explore the parameter space, focusing on the best fits

→ Bayesian inference

\[ P(\text{model}|\text{data}) \propto P(\text{data}|\text{model})P(\text{model}) \propto \exp(-\chi^2/2) \]

- We assume the prior is constant
- Likelihood function
Retrieval techniques

Bayesian inference (MCMC & Nested-sampling)

1) Markov chain Monte Carlo (MCMC)

Ensemble of walkers converging toward best solutions (e.g. Pyrat Bay, Madhusudhan et al.)

2) Nested Sampling

Determination of volumes of equal likelihood (e.g. Tau-Rex, petitRADTRANS, NEMESIS, ARCiS, CHIMERA)

Nested sampling is more efficient to find global maximums of the likelihood
Retrieval techniques

Bayesian inference (MCMC & Nest-sampling)

Intercomparison of retrieval tools for Ariel (Barstow et al. 2022)

Cloudy warm Neptune

Posterior distributions from Tau-Rex

Barstow et al. 2022
Retrieval techniques

Bayesian inference (MCMC & Nest-sampling)

Strength of molecular detection with Bayes factor

<table>
<thead>
<tr>
<th>Planet</th>
<th>Best-fit model</th>
<th>Δlog(E)</th>
<th>Detection</th>
<th>Absorbers (X)</th>
<th>μ (g/mol)</th>
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<tr>
<td>55 Cancri e</td>
<td>2-Active clear</td>
<td>2.94</td>
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<td>HCN</td>
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<tr>
<td>TRAPPIST-1 h</td>
<td>6-Flat-line</td>
<td>none</td>
<td>none</td>
<td>-</td>
<td>2.30</td>
</tr>
</tbody>
</table>

Gressier et al. (submitted)
Retrieval techniques

Bayesian inference (MCMC & Nest-sampling)

JWST imaging of HIP 65426 b from 2-16 micron

Interpolated model grid for MCMC

Interesting for model grids with more than 3 parameters
Retrieval techniques

Optimisation estimation

Method widely used for Earth atmosphere remote sensing and for solar system atmospheres

Principle:
Minimization of a cost function:

\[ J(x) = (y - F(x))^T S_e^{-1} (y - F(x)) + (x - x_a)^T S_a^{-1} (x - x_a) \]

- \( y \) = data vector
- \( x \) = model parameter vector
- \( x_a \) = a priori vector
- \( F(x) \) = forward model

Error covariance matrix

A priori covariance matrix

Weight of the a priori + correlation between parameters

Correlation between temperatures of layer \( i \) and \( j \):

\[ S_{a,ij} = (S_{a,ii} S_{a,jj})^{1/2} e^{-\frac{|\ln \left( \frac{P_i}{P_j} \right)|}{\ell}} \]

Length of correlation

\( \Rightarrow \) Algorithm to iterate toward a state minimizing \( J \)
Retrieval techniques

Optimisation estimation

Exploration of the impact of the a priori TP profile

- A priori profiles can be provided by self-consistent models but their impact must be analysed
- Optimisation estimation is a fast method, ideal for high quality data or with additional constraints

Lee et al. 2011 (NEMESIS)
Optimisation estimation has a great potential for brown dwarfs and young giant planets.
Avantages/disavantages of each method

Chi2 with model grids:
+ Computed just once
- Limited number of free parameters
- Strongly biased by model parametrizations
→ Ideal for limited parameter exploration (i.e. 2D/3D simulations) or low quality dataset

Bayesian inference (MCMC & Nested Sampling):
+ Better estimation of uncertainties than Chi2 maps and shows correlations
+ Exploration a large parameter space
+ Model selection (Bayes factor)
- Not efficient for retrieving profiles
→ Ideal for most cases for exoplanets

Optimisation estimation:
+ Efficient for retrieving profiles
+ Faster than Bayesian inference
- Requires a priori
- Limited exploration of possible solutions
→ Ideal for emission spectroscopy with high-quality dataset and additional constraints

A combination of methods/models can be very useful
Lessons from models and retrieval

1) Atmospheric models work!

TYC 8998-760-1 b

HR8799 b

Zhang et al. (2022)
Petrus et al. in prep
Lessons from models and retrieval

1) Atmospheric models work!

Two main reasons for cases for which models do not work: missing physical processes or 3D effects

Exemple #1: H⁺ in ultra-hot Jupiters

![Graph showing H⁺ in ultra-hot Jupiters with data from Parmentier et al. (2018)]

Exemple #2: nightside clouds on hot Jupiters

![Graph showing nightside clouds on hot Jupiters with data from Gao et al. (2020)]

Parmentier et al. (2018)

Lee et al. (2016)
Lessons from models and retrieval

2) Relative vs absolute measurements

\[ \frac{[\text{CO}]}{[\text{H}_2\text{O}]+[\text{CO}]} = \frac{1}{[\text{H}_2\text{O}] + 1} \]  
(warm planets)

\[ \rightarrow \text{relative measurement} \]

metallicity \( \approx \frac{[\text{H}_2\text{O}]}{[\text{H}_2\text{O}]_{\text{solar}}} \)

\[ \rightarrow \text{absolute measurement} \]

Correlation between \([\text{H}_2\text{O}]\) and \([\text{CO}]\): C/O must be derived directly as posterior

Uncertainty on C/O smaller than on metallicity

Relative measurements are more accurate than absolute measurements
Lessons from models and retrieval

2) Relative vs absolute measurements

Comparison of retrieved parameters (BT-Settl & Exo-REM grids) for brown dwarfs observed with X-Shooter (R=4000)

Removing the continuum + renormalisation can improve the C/O determination at medium/high resolution (elimination of biases from models and observations)
Lessons from models and retrieval

3) Biases: metallicity/cloud

Fortney et al. 2022

Line & Seager 2013
Lessons from models and retrieval

3) Biases: metallicity/cloud

How to probe cloudy/hazy atmospheres?

1) Large spectral coverage (JWST, Ariel)

2) High-resolution spectroscopy (VLT-CRIRES, SPIRou, ELT-ANDES)

3) Thermal phase curves (JWST, Ariel)
4) Biases: thermal inversion

- Detection of a stratospheric thermal inversion on HD209458 b from Spitzer eclipses (Knutson et al. 2007)
- Two classes of hot Jupiters with a transition at $T_{\text{day}} \sim 1600$ K (Fortney et al. 2008)
- Thermal inversion ruled-out after reanalysis of Spitzer data (Diamond-Lowe et al. 2014)
Lessons from models and retrieval

4) Biases: thermal inversion

Evolution of the water feature in eclipses

Mansfield et al. (2021)
Lessons from models and retrieval

4) Biases: thermal inversion

Thermal inversions appear at higher temperatures ($T_{\text{day}} \sim 2000$ K) than initially thought.
Lessons from models and retrieval

4) Biases: isothermal/clouds

Reddening of L dwarfs

HD206893 b’s spectrum = almost a black body

Delorme et al. (2017)
Lessons from models and retrieval

4) Biases: isothermal/clouds

How to reduce spectral features in emission spectra?
Lessons from models and retrieval

4) Biases: isothermal/clouds

How to reduce spectral features in emission spectra?

Charnay et al. (2018)

Tremblin et al. (2017)
Lessons from models and retrieval

4) Biases: isothermal/clouds

Atmospheric retrieval of two L dwarfs by Burningham et al. (2017):

Both clouds + reduced thermal gradient!

But the retrieval might be biased by its relatively simple cloud model
Lessons from models and retrieval

4) Biases: isothermal/clouds

How to break degeneracies between clouds and reduced thermal gradient?

1) Cloud absorption features

2) Thermal evolution

Excess of BD at the LT transition

But:
1) Clouds can be a mixture of species (e.g. Jupiter’s clouds)
2) Best et al. 2020 found a minimum of BD at the LT transition

Silicate feature on VHS 1256 b

Miles et al. (2022)
Lessons from models and retrieval

5) Biases: 3D structure

Inhomogeneous cloud cover

Cloud-free evening terminator

Cloudy morning terminator

Retrieval with cloud fraction:
Degeneracy between clouds and metallicity → need measurements of Rayleigh slope or HR spectroscopy

Line & Parmentier (2016)
Lessons from models and retrieval

5) Biases: 3D structure

- Day-night chemical heterogeneities

- Measurements of C/O can be biased by chemical heterogeneities
- Chemical disequilibrium limits heterogeneities except at high temperature
- Phase curves can be used to map horizontal variations

Pluriel et al. (2020)

Pluriel et al. (2022)
Lessons from models and retrieval

5) Biases: 3D structure

Retrieval of a simulated Ariel phase curve of WASP-43b with nightside clouds

Transit (nightside)

Eclipse (dayside)

Charnay et al. (2021)
Lessons from models and retrieval

5) Biases: 3D structure

Improvement with global/2D retrieval
(e.g. Chubb & Min 2022, Irwin et al. 2019, Changeat et al. 2021)

Chubb & Min (2019)
Lessons from models and retrieval

6) Biases: time-variability

Variability of a brown dwarf with Spitzer

3D simulation of K2-18 b with water clouds

Possible variability of cloudiness and spectra

Apai et al. (2017)

Charnay et al. (2021)
Take-home messages

- We are now in the golden age of exoplanet atmospheres!

- The different atmospheric models (self-consistent/parametric) and retrieval methods (grids, MCMC, optimisation estimation) have advantages and disadvantages. **Ideally, use a combinaison of models**

- **Atmospheric models work** (at least at first order)!

- When models do not work, generally a physical process is missing or it is due to 3D effects

- The interpretation of retrieval outputs is necessary and requires **to understand the potential biases**

- **Clouds/hazes and 3D effects** are likely the largest sources of uncertainties and biases in atmospheric retrievals